Relative Performance Evaluation of Energy Supply Systems Under Uncertain Energy Demands

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Abstract:

In designing energy supply systems, several alternatives for design specifications are proposed, and their performances are evaluated and compared in terms of some criteria such as annual total cost, primary energy consumption, and CO_2 emission. Although the values of these performance criteria depend on not only design specifications but also energy demands and operational strategies, energy demands are uncertain at the design stage. In this paper, a method of evaluating the relative performance for two energy supply systems under uncertain energy demands is proposed based on a linear model. Uncertain energy demands are expressed by intervals. The minimum and maximum, and consequently their interval of a relative performance criterion for two energy supply systems are evaluated for all the possible energy demands within their intervals. An optimization problem included in this evaluation is formulated as a bilevel programming problem, and it is transformed into a mixed-integer linear programming one by adopting the Karush-Kuhn-Tucker conditions and fractional programming. In addition, a hierarchical optimization method is proposed to solve the problem efficiently. A case study is conducted, and the relative difference, or reduction rate in the annual total cost of a cogeneration system in comparison with a conventional energy supply system is evaluated under uncertain energy demands. The minimum and maximum, and consequently their interval of the reduction rate are evaluated, and the corresponding energy demands and operational strategies are found. The influence of the uncertainty in energy demands on these results is also clarified. Through the case study, the validity and effectiveness of the proposed method are clarified.

Keywords:

Energy supply systems, Uncertainty, Relative performance, Interval analysis, Optimization, Mixed-integer linear programming.

1. Introduction

In designing energy supply systems, several alternatives for design specifications are proposed, and their performances are evaluated and compared in terms of some criteria such as annual total cost, primary energy consumption, and CO_2 emission. The values of these performance criteria depend on not only design specifications but also energy demands and operational strategies. However, many conditions under which energy demands are estimated have some uncertainty at the design stage, which makes it impossible to estimate energy demands precisely. If energy demands are estimated certainly, and the performance criteria are evaluated based on certain energy demands, the values of the performance criteria expected at the design stage may not be attained at the operation stage. This is because the energy demands which arise at the operation stage differ from those estimated at the design stage. Similarly, the differences in the values of the performance criteria among the alternatives such as reductions in annual total cost, primary energy consumption, and

CO_2 emission depend on not only design specifications but also energy demands and operational strategies. Thus, the differences in the values of the performance criteria expected at the design stage may not be obtained at the operation stage. Therefore, designers should consider that energy demands have some uncertainty, evaluate the robustness in the performance criteria against the uncertainty, and design the systems rationally in consideration of the robustness. In addition, designers should evaluate the differences in the values of the performance criteria among the alternatives in consideration of the uncertainty in energy demands.

Verderame et al. reviewed many papers on planning and scheduling under uncertainty in multiple sectors, and reviewed some papers on energy planning [1]. Zeng et al. also reviewed many papers on optimization of energy systems planning under uncertainty [2]. In these review papers, the approaches adopted for optimization of energy systems planning were categorized into three ones: stochastic, fuzzy, and interval programming. However, it is difficult for designers to specify stochastic distribution and fuzzy membership functions for uncertain parameters in the first and second approaches, respectively. From the viewpoint of practical applications, it is much more meaningful for designers to specify fluctuation intervals for uncertain parameters in the third approach. Yokoyama and Ito proposed a robust optimal design method of energy supply systems in consideration of the economic robustness against the uncertainty in energy demands based on the minimax regret criterion [3]. Yokoyama et al. revised this robust optimal design method so that it can be applied to energy supply systems with more complex configurations and larger numbers of periods set to consider seasonal and hourly variations in energy demands [4]. Assavapokee et al. presented a general framework for the robust optimal design based on the minimax regret criterion [5]. Yokoyama et al. also proposed a method of comparing two energy supply systems under uncertain energy demands by utilizing a part of the revised robust optimal design method [4]. However, this method cannot evaluate the relative differences but the absolute differences in the values of the performance criteria, which means that although the method can evaluate the reductions in the values of the performance criteria, it cannot evaluate the reduction rates. On the other hand, Yokoyama and Ito also proposed a robust optimal design method based on the relative robustness criterion [6]. Assavapokee et al. presented a general framework for the robust optimal design based on the relative robustness criterion [7]. This idea can be applied to the evaluation of the relative differences in the values of the performance criteria.

In this paper, a method of evaluating the relative performance for two energy supply systems under uncertain energy demands is proposed based on a linear model. Uncertain energy demands are expressed by intervals. The minimum and maximum, and consequently their interval of a relative performance criterion for two energy supply systems are evaluated for all the possible energy demands within their intervals. An optimization problem included in this evaluation is formulated as a bilevel programming one, and it is transformed into a mixed-integer linear programming (MILP) one by adopting the Karush-Kuhn-Tucker conditions and fractional programming [8] for the numerator and denominator, respectively, in the problem. Since the denominator relates the variables at all the periods, the problem cannot be solved in a reasonable computation time even by a general commercial MILP solver. Thus, a hierarchical optimization method is proposed to solve the problem efficiently. A case study is conducted, and the relative difference, or reduction rate in the annual total cost of a cogeneration system in comparison with a conventional energy supply system is evaluated under uncertain energy demands. The minimum and maximum, and consequently their interval of the reduction rate are evaluated, and the corresponding energy demands and operational strategies are sought. The influence of the uncertainty in energy demands on these results is also examined. Through the case study, the validity and effectiveness of the proposed method are investigated.

2. Evaluation of relative performance

Fundamentals for a method of evaluating the relative performance for two energy supply systems under uncertain energy demands is presented here. As design specifications, equipment capacities and utility maximum demands $\boldsymbol{x}_{\mathrm{A}}$ and $\boldsymbol{x}_{\mathrm{B}}$ for energy supply systems A and B, respectively, are assumed to be determined *a priori*. The annual total cost is adopted here as the performance criterion for the comparison, although any one may be adopted. It is evaluated by the annualized costs method as the sum of the annual capital cost of equipment, the annual demand charge of purchased utilities, and the annual energy charge of purchased utilities. The annual energy charge is the sum of energy charges at periods on representative days in a typical year. Since the annual total costs f_{A} and f_{B} of systems A and B depend on energy demands $\boldsymbol{y}$ as well as operational strategies $\boldsymbol{z}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{B}}$, respectively, $\boldsymbol{z}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{B}}$ are determined so as to minimize f_{A} and f_{B}, respectively, subject to constraints such as performance characteristics of equipment and energy balance/supply-demand relationships for energy demands $\boldsymbol{y}$. This is because in fact the operational strategies can be adjusted for energy demands which become certain at the operation stage. It is assumed that the annual total costs f_{A} and f_{B} and the constraints are linear with respect to $\boldsymbol{y}$ as well as $\boldsymbol{z}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{B}}$, respectively. It is also assumed that the equipment capacities and utility maximum demands $\boldsymbol{x}_{\mathrm{A}}$ and $\boldsymbol{x}_{\mathrm{B}}$ along with the operational strategies $\boldsymbol{z}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{B}}$, respectively, satisfy the flexibility, or the feasibility in energy supply for all the possible energy demands $\boldsymbol{y}$.

Under the aforementioned conditions, the relative difference r in the annual total cost between systems A and B under certain energy demands $\boldsymbol{y}$ is expressed as

$$r = \left(\min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) - \min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) \right) \bigg/ \min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \tag{1}$$

where Z_{A} and Z_{B} are the regions for all the possible values of $\boldsymbol{z}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{B}}$ of systems A and B, respectively, and depend on $(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y})$ and $(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y})$, respectively, through the constraints. Eq. (1) is also expressed as

$$r = 1 - \min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) \bigg/ \min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \tag{2}$$

or alternatively

$$r = 1 - 1 \bigg/ \left(\min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \bigg/ \min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) \right) \tag{3}$$

The value of r depends on the energy demands $\boldsymbol{y}$. Under uncertain energy demands $\boldsymbol{y}$, therefore, its minimum $\underline{r}$ and maximum $\overline{r}$ are evaluated for all the possible values of $\boldsymbol{y}$, and consequently their interval $[\underline{r}, \overline{r}]$ is obtained. From Eqs. (2) and (3), the minimum and maximum of the relative difference in the annual total cost between systems A and B are expressed as

$$\underline{r} = 1 - \max_{\boldsymbol{y} \in Y} \left(\min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) \bigg/ \min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \right) \tag{4}$$

and

$$\overline{r} = 1 - 1 \bigg/ \max_{\boldsymbol{y} \in Y} \left(\min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \bigg/ \min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) \right) \tag{5}$$

respectively.

3. Solution method

3.1. Reformulation of optimization problem

Equations (4) and (5) include optimization problems with hierarchical operations of maximization and minimization. Thus, these optimization problems result in bilevel programming ones. In addition, there exist the performance criteria for systems A and B in the numerator and denominator, respectively, in Eq. (4), and vice versa in Eq. (5). Since the forms of both the problems are the same, the solution of the optimization problem in only Eq. (4) is sufficient. Therefore, the following optimization problem is investigated here:

$$\max_{\boldsymbol{y}\in Y}\left(\min_{\boldsymbol{z}_{\mathrm{A}}\in Z_{\mathrm{A}}} f_{\mathrm{A}}\left(\boldsymbol{x}_{\mathrm{A}},\,\boldsymbol{y},\,\boldsymbol{z}_{\mathrm{A}}\right)\Big/\min_{\boldsymbol{z}_{\mathrm{B}}\in Z_{\mathrm{B}}} f_{\mathrm{B}}\left(\boldsymbol{x}_{\mathrm{B}},\,\boldsymbol{y},\,\boldsymbol{z}_{\mathrm{B}}\right)\right) \tag{6}$$

First, the operation of minimization with respect to $\boldsymbol{z}_{\mathrm{B}}$ in the denominator is moved forward to reformulate Eq. (6) as

$$\max_{\boldsymbol{y}\in Y}\max_{\boldsymbol{z}_{\mathrm{B}}\in Z_{\mathrm{B}}}\left(\min_{\boldsymbol{z}_{\mathrm{A}}\in Z_{\mathrm{A}}} f_{\mathrm{A}}\left(\boldsymbol{x}_{\mathrm{A}},\,\boldsymbol{y},\,\boldsymbol{z}_{\mathrm{A}}\right)\Big/ f_{\mathrm{B}}\left(\boldsymbol{x}_{\mathrm{B}},\,\boldsymbol{y},\,\boldsymbol{z}_{\mathrm{B}}\right)\right) \tag{7}$$

Then, this problem is reformulated as an ordinary one level optimization one by applying the Karush-Kuhn-Tucker conditions to the minimization problem at the lower level. This reformulation produces the complementarity constraint which includes the inner product of inequality constraint vectors and the corresponding Lagrange multiplier vectors. Then, binary variable vectors and inequality constraints are introduced to linearize the nonlinear terms due to this complementarity condition exactly [9].

In addition, the problem is reformulated into an MILP one by the fractional programming method [8]. On the assumption that the value of the denominator in Eq. (7) is positive, its inverse is replaced with a continuous variable, and the operation variables $\boldsymbol{z}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{B}}$ as well as the energy demands $\boldsymbol{y}$ are replaced with continuous variables which are the products of themselves and the inverse of the denominator.

The aforementioned reformulations convert Eq. (7) to the following equation:

$$\max_{\boldsymbol{y}',\,\boldsymbol{z}_{\mathrm{A}}',\,\boldsymbol{z}_{\mathrm{B}}',\,\boldsymbol{\mu}_{\mathrm{A1}},\,\boldsymbol{\mu}_{\mathrm{A2}},\,\boldsymbol{\lambda}_{\mathrm{A1}},\,\boldsymbol{\lambda}_{\mathrm{A2}},\,\boldsymbol{\delta}_{\mathrm{A1}},\,\boldsymbol{\delta}_{\mathrm{A2}},\,q} F(\boldsymbol{x}_{\mathrm{A}},\,\boldsymbol{x}_{\mathrm{B}},\,\boldsymbol{y}',\,\boldsymbol{z}_{\mathrm{A}}',\,\boldsymbol{z}_{\mathrm{B}}',\,\boldsymbol{\mu}_{\mathrm{A1}},\,\boldsymbol{\mu}_{\mathrm{A2}},\,\boldsymbol{\lambda}_{\mathrm{A1}},\,\boldsymbol{\lambda}_{\mathrm{A2}},\,\boldsymbol{\delta}_{\mathrm{A1}},\,\boldsymbol{\delta}_{\mathrm{A2}},\,q) \tag{8}$$

where F is the converted objective function to be maximized. The arguments are the variables to be determined, and their regions are omitted here because they are related with one another. Here, $\boldsymbol{\mu}_{\mathrm{A1}}$ and $\boldsymbol{\mu}_{\mathrm{A2}}$, and $\boldsymbol{\lambda}_{\mathrm{A1}}$ and $\boldsymbol{\lambda}_{\mathrm{A2}}$ are Lagrange multiplier vectors corresponding to the equality and inequality constraint vectors, respectively, in the numerator in Eq. (7). $\boldsymbol{\delta}_{\mathrm{A1}}$ and $\boldsymbol{\delta}_{\mathrm{A2}}$ are binary variable vectors introduced to linearize the nonlinear terms due to the complementarity condition. q is a continuous variable for the denominator in Eq. (7), and $\boldsymbol{z}_{\mathrm{A}}'$, $\boldsymbol{z}_{\mathrm{B}}'$, and $\boldsymbol{y}'$ are the products of $\boldsymbol{z}_{\mathrm{A}}$, $\boldsymbol{z}_{\mathrm{B}}$, and $\boldsymbol{y}$, respectively, multiplied by q.

A detailed procedure for reformulating the bilevel programming problem of Eq. (7) to the MILP problem of Eq. (8) is shown in the appendix.

3.2. Hierarchical optimization method

Any general commercial MILP solvers can be applied to solving the optimization problem of Eq. (8). However, the number of binary variables can increase drastically with the complexity of energy supply systems, especially system A, and the number of periods set to consider seasonal and hourly variations in energy demands. In addition, the inverse of the denominator in Eq. (7) q relates the other variables and the constraints at all the periods. These features make it difficult to solve the problem even by general commercial MILP solvers. However, if the value of q is assumed, the other variables and the constraints can be divided into subsets independent at the respective periods. Although the value of q cannot be assumed easily because q is a continuous variable, the range for the value of q may be assumed. Therefore, the following hierarchical optimization method is proposed to solve the problem more efficiently.

The method is composed of the following two hierarchical levels. At the upper level, the branch and bound method is applied to search the range for the value of q where its optimal value exists by adopting q as the unique branching variable. For this purpose, the lower and upper limits for q are determined in advance as follows:

$$\underline{q} = 1 \Big/ \max_{\boldsymbol{y} \in Y} \min_{\boldsymbol{z}_\mathrm{B} \in Z_\mathrm{B}} f_\mathrm{B}\left(\boldsymbol{x}_\mathrm{B},\, \boldsymbol{y},\, \boldsymbol{z}_\mathrm{B}\right) \tag{9}$$

and

$$\overline{q} = 1 \Big/ \min_{\boldsymbol{y} \in Y} \min_{\boldsymbol{z}_\mathrm{B} \in Z_\mathrm{B}} f_\mathrm{B}\left(\boldsymbol{x}_\mathrm{B},\, \boldsymbol{y},\, \boldsymbol{z}_\mathrm{B}\right) \tag{10}$$

respectively. The optimization problem in Eq. (9) is a bilevel linear programming one, and it can be solved by converting it into an ordinary one level MILP problem based on a part of the aforementioned reformulations. The optimization problem in Eq. (10) is an ordinary linear programming one, and it can be solved easily.

The overall range $[\,\underline{q},\ \overline{q}\,]$ for the value of q is divided into its sub-ranges by the branching operation, and the MILP problem is solved in each sub-range. If the MILP problem in a sub-range cannot be solved easily, the sub-range is further divided into smaller ones, and the MILP problem is solved again in each smaller sub-range. If the MILP problem in a sub-range can be solved, and the value of the objective function for its optimal solution is larger than an upper bound for the optimal value of the objective function in the overall range, the incumbent solution and the upper bound in the overall range are replaced with this optimal solution and its value of the objective function, respectively. If the MILP problem in a sub-range is infeasible, the sub-range is fathomed by the bounding operation. Even if the MILP problem in a sub-range cannot be solved easily, if an upper bound for the optimal value of the objective function in the sub-range becomes smaller than the upper bound for the optimal value of the objective function in the overall range while the MILP problem in the sub-range is being solved, it can be judged that there exists no optimal solution in the sub-range, and the sub-range is also fathomed by the bounding operation.

At the lower level, the MILP problems in sub-ranges are solved using a general commercial MILP solver repeatedly. In this paper, CPLEX Ver. 12.6.0.0 is used for this purpose through the modeling system for mathematical programming GAMS Ver. 24.2.1 [10].

4. Case study

4.1. System configurations

The performance of a gas turbine cogeneration system (system A) for district energy supply shown in Fig. 1 is compared with that of a conventional energy supply system (system B) without cogeneration. System A is composed of a gas turbine generator (GT), a waste heat recovery boiler (BW), a gas-fired auxiliary boiler (BG), an electric compression refrigerator (RE), a steam absorption refrigerator (RS), a device for receiving electricity (EP), and a pump for supplying cold water (PC). Electricity is supplied to users by operating the gas turbine generator and purchasing electricity from an outside electric power company. Electricity is also used to drive the electric compression refrigerator, pump, and other auxiliary machinery in the system. Exhaust heat generated from the gas turbine is recovered in the form of steam by the waste heat recovery boiler, and is used for heat supply. An excess of exhaust heat is disposed of through an exhaust gas dumper. A shortage of steam is supplemented by the gas-fired auxiliary boiler. Cold water for space cooling is supplied by the electric compression and steam absorption refrigerators. Steam is used for space heating and hot water supply. On the other hand, system B is composed of a gas-fired auxiliary boiler, an electric compression refrigerator, a steam absorption refrigerator, a device for receiving electricity, and a pump for supplying cold water.

A concrete formulation of the annual total cost as the performance criteria as well as the performance characteristics of equipment and energy balance / supply-demand relationships as the constraints is omitted here.

4.2. Evaluation conditions

A typical year is divided into three representative days, i.e., summer, winter, and mid-season, which have 122, 121, and 122 days per year, respectively. Furthermore, each day is divided into 24 periods each of which has 1 h, and the duration per year of each period is given correspondingly. Averages of electricity, steam, and cold water demands for each period are estimated. Electricity, steam, and cold water demands for each period are assumed to vary within $\pm\alpha$ times of their averages, and correspondingly their upper and lower limits are given. As an example, Fig. 2 shows the averages of electricity, steam, and cold water demands on the representative day in summer. Table 1 shows equipment capacities and utility maximum demands given as design specifications. These values are determined in the case of $\alpha = 0.2$ using the robust optimal design method based on the minimax regret criterion [4]. Other input data are given in Table 2. All values for performance characteristic values of equipment as well as unit costs of equipment and utilities are set based on their real data. In addition, all values for costs are stated in yen, which is equivalent to about 8.5×10^{-3} dollars and 7.4×10^{-3} euro on the recent exchange rate. The capital recovery factor is set by assuming that the life of equipment and the interest rate are 15 y and 0.1, respectively.

4.3. Results and discussion

The minimum and maximum of the relative difference in the annual total cost between the cogeneration system (system A) and conventional energy supply system (system B) are evaluated by the proposed method. Figure 3 shows the relationship between the uncertainty in energy demands α, and the minimum and maximum of the relative difference in the annual total cost. In addition, the relative differences in the annual total cost in case that average, maximum, and minimum energy demands are selected at all the periods are also included in Fig. 3. With an increase in α, the minimum of the relative difference decreases, while the maximum increases, and consequently their interval increases. In addition, the decreasing rate in the minimum of the relative difference increases slightly, while the increasing rate in the maximum of the relative difference decreases slightly. As a result, the averaged relative difference decreases slightly. This is because the cogeneration system becomes disadvantageous for unbalanced electricity and heat demands with an increase in α. The relative difference in case that the minimum energy demands are selected is smaller than that in case that the average ones are selected, and is much larger than the minimum of the relative difference. On the other hand, the relative difference in case that the maximum energy demands are selected is slightly larger than that in case that the average ones are selected, and is much smaller than the maximum of the relative difference. These results show that it is difficult to

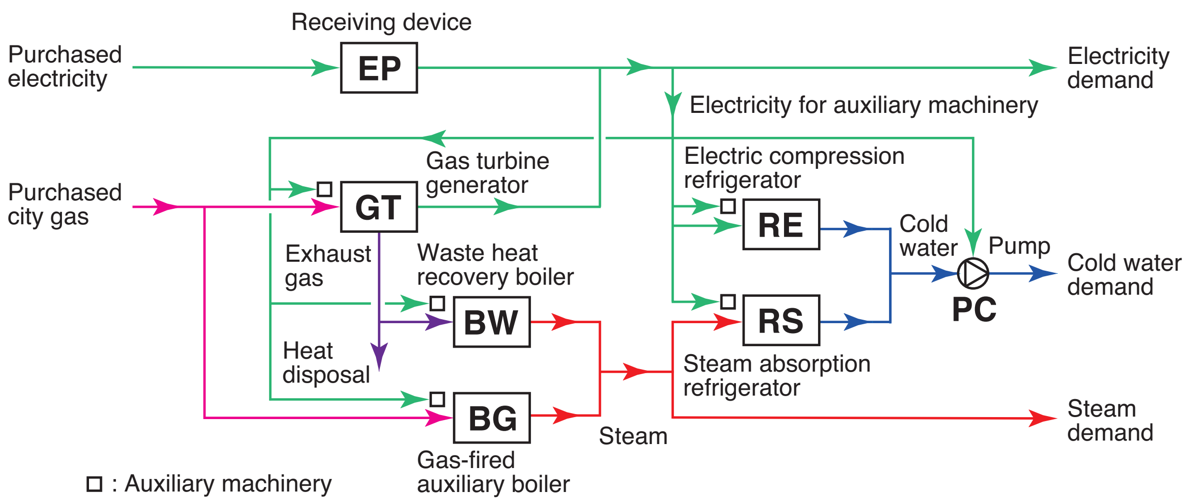


Fig. 1. Configuration of gas turbine cogeneration system (system A)

determine the interval of the relative difference by a conventional sensitivity analysis where energy demands are changed simultaneously and similarly at all the periods.

As examples, Figs. 4 and 5 show the energy demands in summer which give the minimum and maximum, respectively, of the relative difference in the annual total cost in the case of α = 0.2. In these figures, dot-dash lines show the average energy demands, and solid lines and marks show the energy demands which give the minimum and maximum of the relative difference. In Fig. 4, the electricity demands at the upper limits are selected, while the steam and cold water demands at the lower limits are selected, during the daytime. This is because these electricity and heat demands are unbalanced, and are disadvantageous to the cogeneration system, which makes the relative dif-

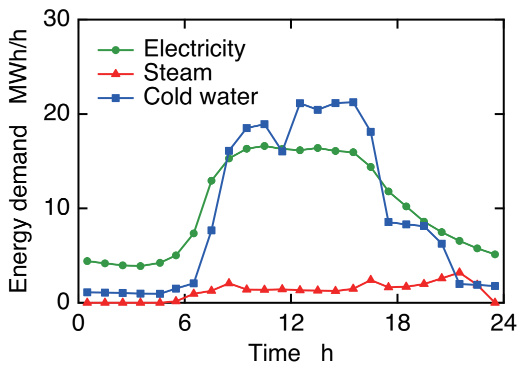


Fig. 2. Average energy demands in summer

Table 1. Equipment capacities and utility maximum demands

Equipment/utility		System A	System B
Gas turbine generator	MW	9.79	—
Waste heat recovery boiler	MW	15.86	—
Gas-fired auxiliary boiler	MW	15.61	26.12
Electric compression refrigerator	MW	1.30	1.39
Steam absorption refrigerator	MW	24.19	24.10
Electricity	MW	12.62	22.02
Natural gas	m^3/h	3.51	2.51

Table 2. Other input data

Performance characteristic values of equipment		
Gas turbine generator	Electricity	3.23 kW/(m^3/h)
	Exhaust heat	6.71 kW/(m^3/h)
Waste heat recovery boiler		0.78 kW/kW
Gas-fired auxiliary boiler		10.40 kW/(m^3/h)
Electric compression refrigerator		5.04 kW/kW
Steam absorption refrigerator		1.25 kW/kW
Electricity consumptions for auxiliary machinery		
Gas turbine generator		0.121 kW/(m^3/h)
Waste heat recovery boiler		0.005 kW/kW
Gas-fired auxiliary boiler		0.051 kW/(m^3/h)
Electric compression refrigerator		0.215 kW/kW
Steam absorption refrigerator		0.079 kW/kW
Pump		0.025 kW/kW
Capital unit costs of equipment		
Gas turbine generator		230.0×10^3 yen/kW
Waste heat recovery boiler		9.6×10^3 yen/kW
Gas-fired auxiliary boiler		6.9×10^3 yen/kW
Electric compression refrigerator		46.7×10^3 yen/kW
Steam absorption refrigerator		43.7×10^3 yen/kW
Receiving device		56.0×10^3 yen/kW
Unit costs for demand charge of utilities		
Electricity		1.74×10^3 yen/(kW·month)
Natural gas		2.37×10^3 yen/(m^3/h·month)
Unit costs for energy charge of utilities		
Electricity		11.0 yen/kWh
Natural gas		31.0 yen/m^3
Parameters for annual total cost		
Capital recovery factor		0.132
Interest rate		0.10
Ratio of salvage value to capital cost		0.0

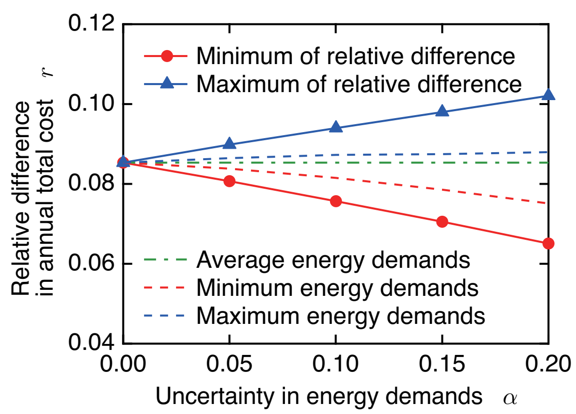


Fig. 3. Relationship between uncertainty in energy demands, and minimum and maximum of relative difference in annual total cost

ference minimum. In Fig. 5, on the other hand, the electricity demands at the lower limits are selected, while the steam demands at the upper limits are selected, and the cold water demands are almost proportional to the electricity supplies by the cogeneration unit as shown below, during the daytime. These energy demands are very advantageous to the cogeneration system, because the heat to power ratio of energy demands almost coincides with the heat to power ratio of energy supplies by the cogeneration unit.

Figures 6 and 7 show the optimal operational strategies of the cogeneration system corresponding to the energy demands in summer which give the minimum and maximum, respectively, of the relative difference in the annual total cost in the case of $\alpha = 0.2$. Figures (a) to (c) show the load allocations for electricity, steam, and cold water supplies, respectively. Although the cogeneration system is operated at the rated load state during the daytime in both Figs. 6 (a) and 7 (a), the amount of purchased electricity in Fig. 6 (a) is much larger than that in Fig. 7 (a), because the electricity demands at the upper and lower limits are selected in Figs. 4 and 5, respectively. Thus, the amount of exhaust heat which is available by operating the gas turbine generator is the same in both Figs. 6 (b) and 7 (b). However, since the heat demands at the lower limits are selected in Fig. 4, a large amount of exhaust heat must be disposed of in Fig. 6 (b). On the other hand, since the heat demands are almost proportional to the electricity supplies by the cogeneration unit, exhaust heat is utilized more effectively in Fig. 7 (b).

5. Conclusions

A method of evaluating the relative performance for two energy supply systems under uncertain energy demands has been proposed based on a linear model. The minimum and maximum, and consequently their interval of a relative performance criterion for two energy supply systems have been evaluated for all the possible energy demands. An optimization problem included in this evaluation has been formulated as a bilevel programming one, and it has been transformed into an MILP one. Then, a hierarchical optimization method has been proposed to solve the problem efficiently. Finally, a case study has been conducted, and the relative difference or reduction rate in the annual total cost of a cogeneration system in comparison with a conventional energy supply system has been evaluated under uncertain energy demands. Through the case study, the following main results have been obtained:

- The minimum of the relative difference is much smaller than the relative difference in case that the minimum energy demands are selected at all the periods. On the other hand, the maximum of the relative difference is much larger than the relative difference in case that the maximum energy demands are selected at all the periods.
- When the relative difference has its minimum, the electricity demands at the upper limits are selected, while the heat demands at the lower limits are selected during the daytime. On the

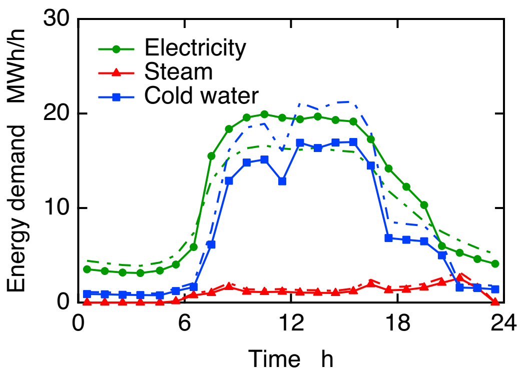


Fig. 4. Energy demands in summer for minimum of relative difference in annual total cost ($\alpha = 0.2$)

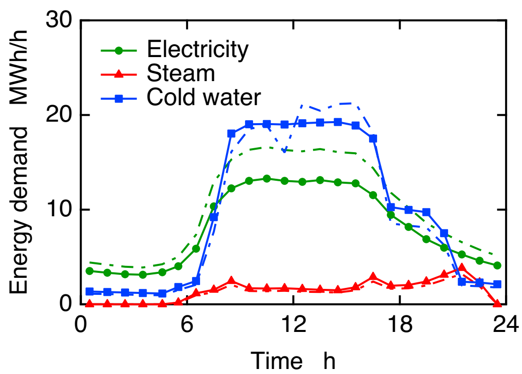
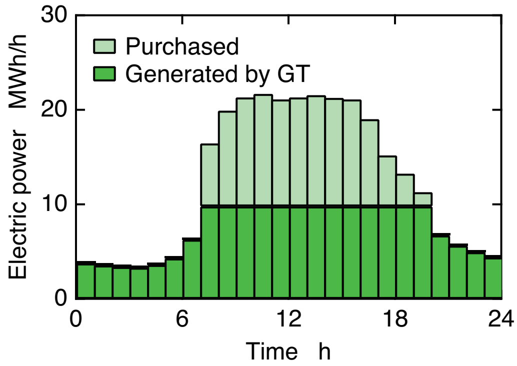


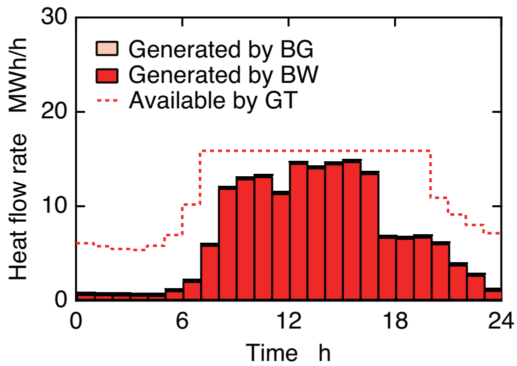
Fig. 5. Energy demands in summer for maximum of relative difference in annual total cost ($\alpha = 0.2$)

other hand, when the relative difference has its maximum, the electricity demands at the lower limits are selected, while the heat demands are almost proportional to the electricity supplies by the cogeneration unit during the daytime.

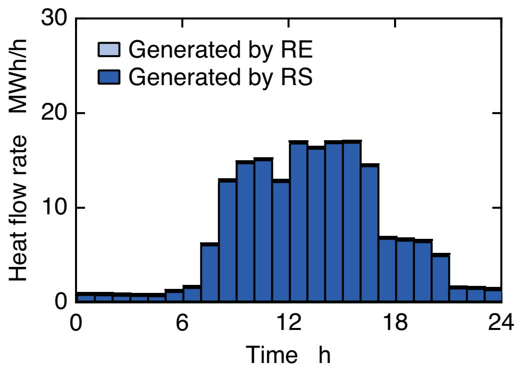
- With an increase in the uncertainty in energy demands, the minimum of the relative difference decreases, while the maximum increases, and consequently their interval increases. In addition, the decreasing rate in the minimum of the relative difference increases slightly, while the increasing rate in the maximum of the relative difference decreases slightly. As a result, the averaged relative difference decreases slightly.
- These features on the relative difference depend significantly on the relationship between the heat to power ratio of uncertain energy demands and that of energy supplies by the cogeneration unit.



(*a*) *Load allocation for electricity supply*

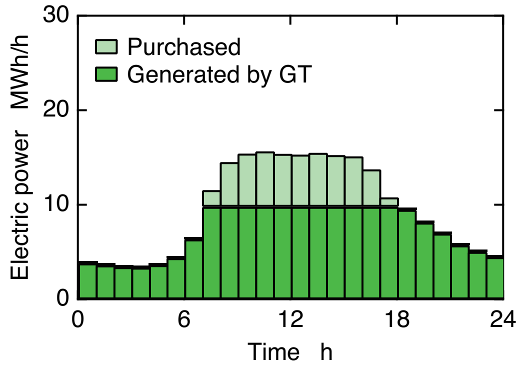


(*b*) *Load allocation for steam supply*

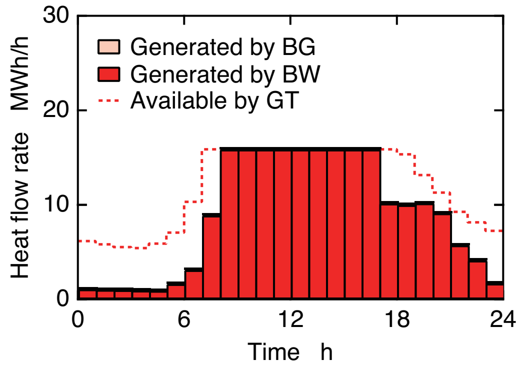


(*c*) *Load allocation for cold water supply*

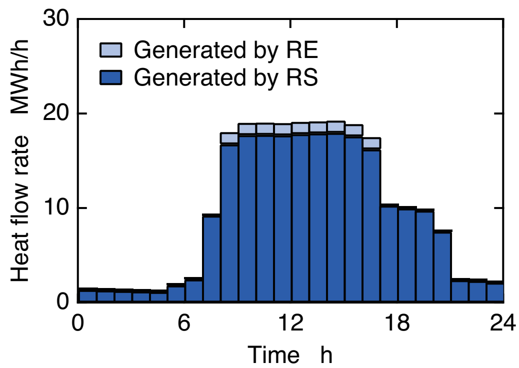
Fig. 6. Optimal operational strategy for energy demands in summer for minimum of relative difference in annual total cost (α = *0.2*)



(*a*) *Load allocation for electricity supply*



(*b*) *Load allocation for steam supply*



(*c*) *Load allocation for cold water supply*

Fig. 7. Optimal operational strategy for energy demands in summer for maximum of relative difference in annual total cost (α = *0.2*)

- It is difficult to obtain these results by a conventional sensitivity analysis where energy demands are changed simultaneously and similarly at all the periods. The results show the validity and effectiveness of the proposed method.

Appendix

A detailed procedure for reformulating the bilevel programming problem of Eq. (7) to the MILP problem of Eq. (8) is shown here.

The numerator and denominator in Eq. (7) are expressed by the following equations:

$$\left.\begin{array}{ll} \min\limits_{\boldsymbol{z}_{\mathrm{A}}} & f_{\mathrm{A}} = \boldsymbol{c}_{\mathrm{A}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{A}} + \boldsymbol{d}_{\mathrm{A}}^{\mathrm{T}}\boldsymbol{z}_{\mathrm{A}} \\ \text{sub. to} & \boldsymbol{A}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}} = \boldsymbol{y} \\ & \boldsymbol{A}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}} = \boldsymbol{b}_{\mathrm{A}} \\ & \boldsymbol{B}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}} \leq \boldsymbol{x}_{\mathrm{A}} \\ & \boldsymbol{B}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}} \geq \boldsymbol{0} \end{array}\right\} \tag{A1}$$

and

$$\left.\begin{array}{ll} f_{\mathrm{B}} = \boldsymbol{c}_{\mathrm{B}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{B}} + \boldsymbol{d}_{\mathrm{B}}^{\mathrm{T}}\boldsymbol{z}_{\mathrm{B}} & \\ \text{sub. to} & \boldsymbol{A}_{\mathrm{B1}}\boldsymbol{z}_{\mathrm{B}} = \boldsymbol{y} \\ & \boldsymbol{A}_{\mathrm{B2}}\boldsymbol{z}_{\mathrm{B}} = \boldsymbol{b}_{\mathrm{B}} \\ & \boldsymbol{B}_{\mathrm{B1}}\boldsymbol{z}_{\mathrm{B}} \leq \boldsymbol{x}_{\mathrm{B}} \\ & \boldsymbol{B}_{\mathrm{B2}}\boldsymbol{z}_{\mathrm{B}} \geq \boldsymbol{0} \end{array}\right\} \tag{A2}$$

respectively, where $\boldsymbol{A}_{\mathrm{A1}}$, $\boldsymbol{A}_{\mathrm{A2}}$, $\boldsymbol{B}_{\mathrm{A1}}$, $\boldsymbol{B}_{\mathrm{A2}}$, $\boldsymbol{A}_{\mathrm{B1}}$, $\boldsymbol{A}_{\mathrm{B2}}$, $\boldsymbol{B}_{\mathrm{B1}}$, and $\boldsymbol{B}_{\mathrm{B2}}$ are coefficient matrices, $\boldsymbol{c}_{\mathrm{A}}$, $\boldsymbol{d}_{\mathrm{A}}$, $\boldsymbol{c}_{\mathrm{B}}$, and $\boldsymbol{d}_{\mathrm{B}}$ are coefficient vectors, $\boldsymbol{b}_{\mathrm{A}}$ and $\boldsymbol{b}_{\mathrm{B}}$ are constant term vectors, and T denotes a transposition of a vector or matrix. The uncertain energy demands are assumed to be constrained by the following intervals:

$$\underline{\boldsymbol{y}} \leq \boldsymbol{y} \leq \overline{\boldsymbol{y}} \tag{A3}$$

where $\overline{\boldsymbol{y}}$ and $\underline{\boldsymbol{y}}$ are upper and lower limits for $\boldsymbol{y}$. Here, the Karush-Kuhn-Tucker conditions are applied to the minimization operation with respect to $\boldsymbol{z}_{\mathrm{A}}$ in the numerator in Eq. (7), or Eq. (A1), which results in

$$\left.\begin{array}{ll} f_{\mathrm{A}} = \boldsymbol{c}_{\mathrm{A}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{A}} + \boldsymbol{d}_{\mathrm{A}}^{\mathrm{T}}\boldsymbol{z}_{\mathrm{A}} & \\ \text{sub. to} & \boldsymbol{A}_{\mathrm{A1}}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{A1}} + \boldsymbol{A}_{\mathrm{A2}}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{A2}} + \boldsymbol{B}_{\mathrm{A1}}\boldsymbol{\lambda}_{\mathrm{A1}} - \boldsymbol{B}_{\mathrm{A2}}\boldsymbol{\lambda}_{\mathrm{A2}} = -\boldsymbol{d}_{\mathrm{A}} \\ & \boldsymbol{A}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}} = \boldsymbol{y} \\ & \boldsymbol{A}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}} = \boldsymbol{b}_{\mathrm{A}} \\ & \boldsymbol{B}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}} \leq \boldsymbol{x}_{\mathrm{A}} \\ & \boldsymbol{B}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}} \geq \boldsymbol{0} \\ & \boldsymbol{\lambda}_{\mathrm{A1}} \geq \boldsymbol{0} \\ & \boldsymbol{\lambda}_{\mathrm{A2}} \geq \boldsymbol{0} \\ & \boldsymbol{\lambda}_{\mathrm{A1}}^{\mathrm{T}}(\boldsymbol{B}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}} - \boldsymbol{x}_{\mathrm{A}}) - \boldsymbol{\lambda}_{\mathrm{A2}}^{\mathrm{T}}(\boldsymbol{B}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}}) = 0 \end{array}\right\} \tag{A4}$$

where $\boldsymbol{\mu}_{\mathrm{A1}}$ and $\boldsymbol{\mu}_{\mathrm{A2}}$ are Lagrange multiplier vectors corresponding to the equality constraint vectors in the second and third lines, respectively, and $\boldsymbol{\lambda}_{\mathrm{A1}}$ and $\boldsymbol{\lambda}_{\mathrm{A2}}$ are Lagrange multiplier vectors corresponding to the inequality constraint vectors in the fourth and fifth lines, respectively, in Eq. (A1). The last equality constraint shows the complementarity one which includes the inner product of the inequality constraint vectors and the corresponding Lagrange multiplier vectors. Although this equality constraint is quadratic, it can be converted to the following linear inequality constraints by introducing binary variable vectors $\boldsymbol{\delta}_{\mathrm{A1}}$ and $\boldsymbol{\delta}_{\mathrm{A2}}$ [9]:

$$\left.\begin{array}{l}\boldsymbol{B}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}}-\boldsymbol{x}_{\mathrm{A}}\geq\underset{\sim}{\boldsymbol{G}}_{\mathrm{A1}}\boldsymbol{\delta}_{\mathrm{A1}}\\ \boldsymbol{B}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}}\leq-\underset{\sim}{\boldsymbol{G}}_{\mathrm{A2}}\boldsymbol{\delta}_{\mathrm{A2}}\\ \boldsymbol{\lambda}_{\mathrm{A1}}\leq\tilde{\boldsymbol{\Lambda}}_{\mathrm{A1}}(\mathbf{1}-\boldsymbol{\delta}_{\mathrm{A1}})\\ \boldsymbol{\lambda}_{\mathrm{A2}}\leq\tilde{\boldsymbol{\Lambda}}_{\mathrm{A2}}(\mathbf{1}-\boldsymbol{\delta}_{\mathrm{A2}})\end{array}\right\}\tag{A5}$$

where $\underset{\sim}{\boldsymbol{G}}_{\mathrm{A1}}$ and $\underset{\sim}{\boldsymbol{G}}_{\mathrm{A2}}$ are diagonal matrices with lower bounds for the corresponding inequality constraint vectors as all the diagonal components, $\tilde{\boldsymbol{\Lambda}}_{\mathrm{A1}}$ and $\tilde{\boldsymbol{\Lambda}}_{\mathrm{A2}}$ are diagonal matrices with upper bounds for the corresponding Lagrange multiplier vectors as all the diagonal components, and $\mathbf{1}$ is a vector with 1 as all the components. Thus, the numerator in Eq. (7), or Eq. (A1) results in a mixed-integer linear form.

The fractional programming method is applied to reformulating the denominator in Eq. (7) [8]. The inverse of the denominator in Eq. (7), or the annual total cost of system B in Eq. (A2), is replaced by a continuous variable q as follows:

$$q=1\Big/f_{\mathrm{B}}\tag{A6}$$

This replacement generates quadratic terms as the products of $\boldsymbol{z}_{\mathrm{A}}$, $\boldsymbol{z}_{\mathrm{B}}$, and $\boldsymbol{y}$ multiplied by q in Eqs. (A2) to (A5). However, these quadratic terms are replaced with continuous variable vectors $\boldsymbol{z}_{\mathrm{A}}^{'}$, $\boldsymbol{z}_{\mathrm{B}}^{'}$, and $\boldsymbol{y}'$ as follows:

$$\left.\begin{array}{l}\boldsymbol{z}_{\mathrm{A}}^{'}=\boldsymbol{z}_{\mathrm{A}}q\\ \boldsymbol{z}_{\mathrm{B}}^{'}=\boldsymbol{z}_{\mathrm{B}}q\\ \boldsymbol{y}'=\boldsymbol{y}q\end{array}\right\}\tag{A7}$$

As a result, the ratio of Eq. (A1) to Eq. (A2) and Eq. (A3) are expressed by the following final form:

$$\left.\begin{array}{ll}\multicolumn{2}{l}{\boldsymbol{c}_{\mathrm{A}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{A}}q+\boldsymbol{d}_{\mathrm{A}}^{\mathrm{T}}\boldsymbol{z}_{\mathrm{A}}^{'}}\\ \text{sub. to} & \boldsymbol{A}_{\mathrm{A1}}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{A1}}+\boldsymbol{A}_{\mathrm{A2}}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{A2}}+\boldsymbol{B}_{\mathrm{A1}}\boldsymbol{\lambda}_{\mathrm{A1}}-\boldsymbol{B}_{\mathrm{A2}}\boldsymbol{\lambda}_{\mathrm{A2}}=-\boldsymbol{d}_{\mathrm{A}}\\ & \boldsymbol{A}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}}^{'}=\boldsymbol{y}'\\ & \boldsymbol{A}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}}^{'}=\boldsymbol{b}_{\mathrm{A}}q\\ & \boldsymbol{B}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}}^{'}\leq\boldsymbol{x}_{\mathrm{A}}q\\ & \boldsymbol{B}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}}^{'}\geq\mathbf{0}\\ & \boldsymbol{\lambda}_{\mathrm{A1}}\geq\mathbf{0}\\ & \boldsymbol{\lambda}_{\mathrm{A2}}\geq\mathbf{0}\\ & \boldsymbol{B}_{\mathrm{A1}}\boldsymbol{z}_{\mathrm{A}}^{'}-\boldsymbol{x}_{\mathrm{A}}q\geq\underset{\sim}{\boldsymbol{G}}_{\mathrm{A1}}^{'}\boldsymbol{\delta}_{\mathrm{A1}}\\ & \boldsymbol{B}_{\mathrm{A2}}\boldsymbol{z}_{\mathrm{A}}^{'}\leq-\underset{\sim}{\boldsymbol{G}}_{\mathrm{A2}}^{'}\boldsymbol{\delta}_{\mathrm{A2}}\\ & \boldsymbol{\lambda}_{\mathrm{A1}}\leq\tilde{\boldsymbol{\Lambda}}_{\mathrm{A1}}(\mathbf{1}-\boldsymbol{\delta}_{\mathrm{A1}})\\ & \boldsymbol{\lambda}_{\mathrm{A2}}\leq\tilde{\boldsymbol{\Lambda}}_{\mathrm{A2}}(\mathbf{1}-\boldsymbol{\delta}_{\mathrm{A2}})\\ & \boldsymbol{c}_{\mathrm{B}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{B}}q+\boldsymbol{d}_{\mathrm{B}}^{\mathrm{T}}\boldsymbol{z}_{\mathrm{B}}^{'}=1\\ & \boldsymbol{A}_{\mathrm{B1}}\boldsymbol{z}_{\mathrm{B}}^{'}=\boldsymbol{y}'\\ & \boldsymbol{A}_{\mathrm{B2}}\boldsymbol{z}_{\mathrm{B}}^{'}=\boldsymbol{b}_{\mathrm{B}}q\\ & \boldsymbol{B}_{\mathrm{B1}}\boldsymbol{z}_{\mathrm{B}}^{'}\leq\boldsymbol{x}_{\mathrm{B}}q\\ & \boldsymbol{B}_{\mathrm{B2}}\boldsymbol{z}_{\mathrm{B}}^{'}\geq\mathbf{0}\\ & \underline{\boldsymbol{y}}q\leq\boldsymbol{y}'\leq\overline{\boldsymbol{y}}q\end{array}\right\}\tag{A8}$$

where $\underset{\sim}{G}'_{\mathrm{A1}}$ and $\underset{\sim}{G}'_{\mathrm{A2}}$ are diagonal matrices for lower bounds for the corresponding inequality constraint vectors as all the diagonal components, and are calculated as the products of $\underset{\sim}{G}_{\mathrm{A1}}$ and $\underset{\sim}{G}_{\mathrm{A2}}$, respectively, multiplied by the upper limit $\overline{q}$ of Eq. (10) for the continuous variable q as follows:

$$\left.\begin{aligned} \underset{\sim}{G}'_{\mathrm{A1}} &= \underset{\sim}{G}_{\mathrm{A1}}\overline{q} \\ \underset{\sim}{G}'_{\mathrm{A2}} &= \underset{\sim}{G}_{\mathrm{A2}}\overline{q} \end{aligned}\right\} \tag{A9}$$

Therefore, Eq. (7) results in the mixed-integer linear form of Eq. (8).

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