New features to Barbaro's heat exchanger network algorithm: heat exchanger technologies and waste heat flow representation

Cong-Toan Tran [a]***, Fabien Thibault*** [a]***, Hubert Thieriot*** [a]***, Assaad Zoughaib*** [a] ***and Sandrine Pelloux-Prayer*** [b]

[a] *Centre for Energy efficiency of Systems, MINES ParisTech, PSL, 60 blv St Michel, 75272, Paris, France*
[b] *EPI Department, EDF R&D, Avenue des Renardières – Ecuelles, 77818 Moret sur Loing, France*

Abstract

To improve heat recovery and reduce energy consumption in industrial processes, heat exchanger network (HEN) design has been widely applied, but it has proven to be a challenging task. Plenty of different design methods have been developed since the early 80's. Most of them consider HEN design as a mathematical optimisation problem to be solved. Using existing design methods does not allow taking into account some technical constraints leading to non directly applicable solutions. In this paper, we introduce new features to a recent HEN algorithm: multiple heat exchanger technologies and flexible streams. By considering different heat exchanger technologies since the HEN design step, we will be able to handle heat flows' physical properties and particularities such as corrosion or fouling aspect. Moreover, from an economical point of view, considering different technologies allows having a more accurate cost evaluation depending on heat exchanger configuration (shell and tube, plate heat exchangers, finned tubes...) and materials used for fabrication. Furthermore, the creation of a new stream type with floating outlet temperature will ease realistic heat recovery on waste heat (such as hot fumes). In this article, the implementation of these proposed features into an existing algorithm is described. Then, a literature case is used to illustrate the relevance of the new proposed features.

Keywords

Heat Exchanger Network, Energy Integration, Heat exchanger technologies, MILP

1. Introduction

In industrial process synthesis, the heat exchanger network synthesis (HENS) is the essential step. For the complete update on the HENS methodologies developed over the decades, the reader may consult the recent reviews on the topic, such as the papers [1], [2]. Among the existing methodologies, the method based on the pinch analysis is considered as the most basic one. It has been successfully applied in a large number of process synthesis projects over the world. However, it implies a manual calculation procedure, so difficult to be used for a complex system. In addition, there is no way to ensure that the solution found is the optimal one. These limitations require developing alternative methodologies, such as mathematical programming approaches.

In the mathematical programming approaches, the methods can be classified as either sequential technique or simultaneous technique. While the first technique is based on the strategy that divides the HENS problem into a number of stages of calculation and generally uses a temperature partition, the simultaneous technique aims to find the optimal solution without decomposition of the problem. The simultaneous methods are based on mixed integer non linear programming (MINLP) formulations, which can raise a major difficulty related to the numerical resolution. Indeed, in the case of a non-convex problem, the solution found is probably not the global optimum, but only the local one.

One of the first model using the sequential technique was presented in the paper [3]. After this initiative work, many models have been developed in order to design a more realistic heat exchanger network. Indeed, in almost all existing models a number of assumptions have been made such as isothermal mixing, no split stream and no stream by pass, which allows reducing the complexity of the problem. Among the recent works, linear models were presented in [4], then extended in [5]. Specifically, these models allow approximating the heat exchanger areas, to implicitly determine flow rates in splits, to handle non isothermal mixing and to permit multiple matches between two streams. A real plant layout associated with the space constraints can be also considered during the heat exchanger design [6]. In addition, because the HENS is known as an NP-Hard problem [7], some efforts in managing the computation time have been made [8], [9].

In this context, the objective of our paper is to extend an existing model developed by Barbarro and Bagajewicz [5], in order to provide new functionalities to a designer and limit the required assumptions. Specifically, our model introduces the following features:

- It takes into account multiple heat exchanger (HEX) technologies. In real life, a great number of HEX technologies with different performances and costs exist. In addition, there is a number of constraints related to the use of HEX technologies due to the fact that the designer usually wants to impose a specific technology according to the properties of the streams. The proposed functionality allows the model to find the most appropriate technologies from an economical point of view while satisfying the constraints imposed by the designer.
- It considers flexible streams. In most of the developed methodologies throughout the years only two types of stream are considered: process stream and utility stream [1], [10]. For a process stream the inlet and outlet temperatures and the mass flow rate are fixed. For a utility stream, only the temperatures are fixed, while the flow rate varies to satisfy the hot and cold requirements of the process streams. In most real processes a 3rd type of stream can be identified, and it is referred to as flexible stream in this paper. A flexible stream has an inlet temperature and a fixed mass flow rate, but its outlet temperature can vary. As an example, exhaust streams of a process, like hot fumes, are basically flexible streams because they can be either used for the heat recovery purpose or, under specific conditions, directly released in the environment. Our model enables to handle flexible streams. Specifically, the algorithm will determine the optimum outlet temperature and the heat exchanger network associated.

2. Mathematical model

2.1. Base model

This parts sums up the base model which is the one presented in [5]. For the sake of simplicity, we present only the essential equations. Additional equations will be necessary in some specific conditions, for example when more than one match is permitted between two streams or when non isothermal mixing is allowed. All these formulations are described in [5].

2.1.1. Set definitions

It is necessary to define a number of different sets that will be used through the model. First, a set of zones is defined, namely Z = {z|z is a heat transfer zone}. The use of zones aims at separating the design in different sub-networks that are not interrelated, allowing simplifying the problem complexity. As an example, if the designer wants to respect the rules of thumb based on the pinch method design (without heat transfer across the pinch point), two zones have to be defined (above and below the pinch temperature). This functionality is particularly interesting for a problem having a large number of streams, knowing that the HEN design is an NP-Hard problem [7], so that the calculation time increases exponentially with the number of variables.

The following sets are used to identify hot, cold streams and utilities. Note that a set of stream includes process and utility streams.

H^z = {i|i is a hot stream present in zone z}

C^z = {j|j is a cold stream present in zone z}

HU^z = {i |i is a heating utility present in zone z } ($HU^z \subset H^z$)

CU^z = {j |j is a cooling utility present in zone z } ($CU^z \subset C^z$)

The temperature scale is divided, in each zone, into different intervals. This step allows to perform the heat balances and the area calculations via linear equations. In addition, a shift of $\Delta T_{\min}$ is performed over all cold stream temperatures to guarantee the heat transfer feasibility. We call T_m^U and T_m^L as upper and lower temperatures of interval m. Moreover, different sets related to the temperature intervals are needed to be defined:

M^z = {*m*|*m* is a temperature interval in zone *z*}

M_i^z = {*m*|*m* is a temperature interval belonging to zone *z*, in which hot stream *i* is present}

N_j^z = {*n*|*n* is a temperature interval belonging to zone *z*, in which cold stream *j* is present}

H_m^z = {*i*|*i* is a hot stream present in temperature interval *m* in zone *z*}

C_n^z = {*j*|*j* is a cold stream present in temperature interval *n* in zone *z*}

Finally, the next sets allow the designer to set different constraints, according to his own preference.

P = {(*i*, *j*)| a heat exchange match between hot stream *i* and cold stream *j* is permitted}

P_{im}^H = {*i*| heat transfer from hot stream *i* at interval *m* to cold stream *j* is permitted}

P_{jn}^C = {*j*| heat transfer from hot stream *i* to cold stream *j* at interval *n* is permitted}

NI^H = {*i*| non-isothermal mixing is permitted for hot stream *i*}

NI^C = {*j*| non-isothermal mixing is permitted for cold stream *j*}

The sets P, P_{im}^H and P_{jn}^C are used to either permit or forbid specific heat exchange matches. The sets NI^H and NI^C allow the designer to specify whether non-isothermal mixing of stream splits is permitted.

2.1.2. Heat balance equations

The model is based on transshipment/transportation scheme. Fig. 1 shows an example of heat exchanger between hot stream i and cold stream j, in zone z. $q_{im,jn}^z$ represents heat transportation from interval m of hot stream i to interval n of cold stream j.

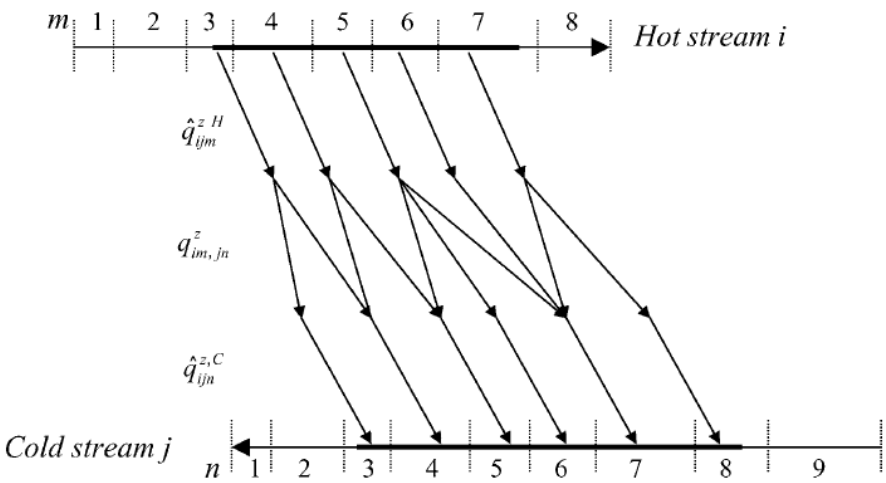

Fig. 1. Basic scheme of the transportation/transshipment model [5]

The heat balance equations state that the heat available on each hot streams or the heat demand of cold streams is equal to the heat transferred to the specific intervals. The following equations are used for hot utilities and hot process streams, respectively. The balance equations for the cold utility and cold process streams can be obtained in a similar way.

$$F_i^H\left(T_m^U - T_m^L\right) = \sum_{\substack{n\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z \qquad z\in Z; m\in M^z; i\in H_m^z; i\in HU^z \tag{1}$$

$$\Delta H_{im}^{z,H} = \sum_{\substack{n\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z \qquad z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; i\notin NI^H \tag{2}$$

Notice that for process streams the enthalpy variations $\Delta H_{im}^{z,H}$ is considered as parameter and can be easily calculated because the flow rates and temperature intervals are known. On the contrary, for utilities the flow rates F_i^H are considered as variable and will be determined by the model.

The required heat transfer area for heat exchange between hot stream i and cold stream j is calculated as:

$$A_{ij}^z = \sum_{m\in M_i^z} \sum_{\substack{n\in N_j^z; T_n^L < T_m^U \\ j\in P_{im}^H; i\in P_{jn}^C}} \frac{q_{im,jn}^z\left(h_{im}+h_{jn}\right)}{\Delta T_{mn}^{ML} h_{im} h_{jn}} \qquad z\in Z; i\in H^z; j\in C^z; (i,j)\in P \tag{3}$$

where: h_{im} and h_{jn} are film heat transfer coefficients for hot stream i in interval m and cold stream j in interval n, respectively

ΔT_{mn}^{ML} is mean logarithmic temperature difference between intervals m and n

In practice, the heat transfer area of a single exchanger is limited. The designer can set maximal area $A_{ij\max}^z$ for each couple of hot stream i and cold stream j, and the number of required heat exchangers is defined through an integer variable U_{ij}^z as follows:

$$A_{ij}^z \le A_{ij\max}^z U_{ij}^z \tag{4}$$

The objective function is to minimize the annualized total cost, which includes the utility and heat exchanger costs. The cost of a heat exchanger is linearized, including a fixed charge cost and a variable one. The total annual cost is expressed as:

$$\min \quad cost = \sum_{z\in Z}\sum_{i\in HU^z} c_i^H F_i^H \Delta T_i + \sum_{z\in Z}\sum_{j\in CU^z} c_j^C F_j^C \Delta T_j + \sum_{z\in Z}\sum_{i\in H^z}\sum_{\substack{j\in C^z \\ (i,j)\in P}} \left(c_{ij}^F U_{ij}^z + c_{ij}^A A_{ij}^z\right) \tag{5}$$

where: c_i^H and c_j^C are hot and cold utility costs, respectively

ΔT_i and ΔT_j are temperature ranges of hot stream i and cold stream j, respectively

c_{ij}^F is fixed cost related to the number of shells

c_{ij}^A is variable cost related to the heat exchanger surface

Note that, when multiple matches between two streams are permitted, it is possible to add some equations and constraints to determine implicitly flow rates in splits. In this case, an additional term related to the multiple matches is added to the cost function [5].

2.2. Extended model

2.2.1. Determination of temperature intervals

The temperature partition is an essential step to guarantee the linearity of the problem. Because the paper [5] didn't detail how to determine the temperature intervals, we propose here a solution for the temperature partition. In each zone z, the angular points of the grand composite curve (GCC) are considered to create a first set of temperature intervals. Then, the partition undergoes successively the three following steps:

- A maximal temperature step $\Delta T_{\max}^{partition}$ is set by the designer. Any higher temperature interval will be halved until the sub-intervals respect the maximal value.
- The base model presented in part 2.1 requires that each stream has to own one internal interval at least. If a stream doesn't satisfy this condition after the first partition step , it is divided into three equal intervals.
- Following the two steps above, if the total number of intervals is less than a minimum number of intervals set by the designer, the largest intervals are halved to meet the condition.

One can see that the maximal temperature step and minimum number of intervals are the key parameters for the trade-off between speed and accuracy of the algorithm. So, these parameters should fit the studied process.

2.2.2. Multiple HEX technologies

This section describes how different HEX technologies can be introduced in the base model. The latter will be modified, so that the algorithm will find the most appropriate technologies while satisfying the imposed constraints.

For this to happen, the variables related to heat exchangers, such as surface, number of units and cost are given an additional index *t* which characterizes the different HEX technologies. For this purpose, a set of technologies is defined, namely T = {t|t is an available technology}.

The model presented in part 2.1 implicitly supposes that the HEX is counter-current type (3). In most real HEX the flow is a mixture of co-current, counter-current and cross flow. In order to take into account this aspect, a correlation factor $FHEX_t$ is introduced for each technology. This factor, set by the designer, represents the efficiency of the technology compared to the counter HEX one. In other words, the required heat transfer area $A_{1,2}$ between two flows 1 and 2 can be determined as:

$$A_{1,2} FHEX_t = \frac{q_{1,2}\left(h_1 + h_2\right)}{\Delta T^{ML} h_1 h_2} \tag{6}$$

where: $q_{1,2}$ is heat transfer quantity

ΔT^{ML} is mean logarithmic temperature difference between flows 1 and 2

h_1 and h_2 are film heat transfer coefficients for flows 1 and 2, respectively

Equation (3) is rewritten as follows:

$$\sum_{t\in T_{ij}} A_{ijt}^z FHEX_t = \sum_{m\in M_i^z} \sum_{\substack{n\in N_j^z; T_n^L < T_m^U \\ j\in P_{im}^H; i\in P_{jn}^C}} \frac{q_{im,jn}^z\left(h_{im} + h_{jn}\right)}{\Delta T_{mn}^{ML} h_{im} h_{jn}} \qquad z\in Z; i\in H^z; j\in C^z; \left(i,j\right)\in P \tag{7}$$

The algorithm will determine A_{ijt}^z while minimizing the cost function. A non-null surface A_{ijt}^z means that the technology t is used for heat exchange between hot stream i and cold stream j.

For each heat exchanger technology, it is possible to permit or forbid heat exchange match between two streams i and j. For this purpose, we use a new set P_t defined as:

P_t = {(i,j)| a heat exchange match between hot stream i and cold stream j via technology t is permitted}

The next equations aim to satisfy these constraints (8) and guarantee the consistency of heat transfer areas calculated (9).

$$A_{ijt}^z = 0 \qquad z\in Z; i\in H^z; j\in C^z; t\in T; \left(i,j\right)\notin P_t \tag{8}$$

$$A_{ijt}^{z} \geq 0 \qquad z \in Z; i \in H^z; j \in C^z; t \in T; (i,j) \in P_t \tag{9}$$

In place of equation (4), the next equation will be used to calculate the number of heat exchangers matching hot stream i and cold stream j in zone z U_{ijt}^{z} (the maximum shell area $A_{ijt\max}^{z}$ is set by the designer).

$$A_{ijt}^{z} \leq A_{ijt\max}^{z} U_{ijt}^{z} \tag{10}$$

To take into account the cost of different technologies, the total annual cost (5) is rewritten as:

$$\min \quad cost = \sum_{z\in Z}\sum_{i\in HU^z} c_i^H F_i^H \Delta T_i + \sum_{z\in Z}\sum_{j\in CU^z} c_j^C F_j^C \Delta T_j + \sum_{z\in Z}\sum_{i\in H^z}\sum_{\substack{j\in C^z \\ (i,j)\in P}}\sum_{t\in T_{ij}} \left(c_{ijt}^F U_{ijt}^z + c_{ijt}^A A_{ijt}^z \right) \tag{11}$$

2.2.3. Flexible stream

In order to take into account the flexible streams, it is necessary to set, by the designer, the following sets and parameters. For a flexible stream, the inlet temperature is fixed as required for a normal stream, but the outlet temperature varies between the lower and upper values set by the designer.

HF^z = {i|i is a flexible hot stream present in zone z} ($HF^z \subset H^z$)

CF^z = {j|j is a flexible cold stream present in zone z} ($CF^z \subset C^z$)

$\left(T_{i,out}^L, T_{i,out}^U\right)$: temperature range of the outlet temperature of stream i

The part $\left(T_{i,out}^L, T_{i,out}^U\right)$ is called surplus part. The main idea is to consider a flexible stream as a normal stream but we allow heat exchange between the surplus part and additional utilities, namely virtual utilities, without any cost. Specifically, a virtual hot utility is artificially created as a normal hot utility with inlet and outlet temperatures relatively high so that it can satisfy the total heat demand on every cold stream. We use the following definitions:

$$\begin{aligned} T_{i_v,out}^{z} &= \max\left(T_m^U \Big|_{m\in M^z} \right) \\ T_{i_v,in}^{z} &= \max\left(T_m^U \Big|_{m\in M^z} \right) + \frac{3}{2}\Delta T_{\max}^{partition} \end{aligned} \tag{12}$$

where the index i_v refers to the virtual hot utility.

Similarly, a virtual cold utility can be defined as follows:

$$\begin{aligned} T_{j_v,out}^{z} &= \min\left(T_m^L \Big|_{m\in M^z} \right) \\ T_{j_v,in}^{z} &= \min\left(T_m^L \Big|_{m\in M^z} \right) - \frac{3}{2}\Delta T_{\max}^{partition} \end{aligned} \tag{13}$$

where the index j_v refers to the virtual cold utility.

Once the temperatures are defined, the temperature partition step is carried out (part 2.2.1), by using the temperature range $\left(T_{i,out}^L, T_{i,out}^U\right)$ as temperature interval bounds. That allows to define the following sets:

MF_i^z {m|m is a temperature interval belonging to zone z, in which the surplus zone of hot stream i is present} ($MF_i^z \subset M_i^z$)

NF_j^z {n|n is a temperature interval belonging to zone z, in which the surplus zone of cold stream j is present} ($NF_j^z \subset N_j^z$)

The following equations will be used to ensure that the virtual utilities are only allowed to match the surplus parts of the flexible streams.

for the virtual hot utility:

$$q^{z}_{i_v m, jn} = 0 \qquad z \in Z;\ j \notin CF^{z}; n \in N^{z}_{j}; m \in M^{z}_{i_v}$$
$$q^{z}_{i_v m, jn} = 0 \qquad z \in Z;\ j \in CF^{z}; n \in N^{z}_{j} \setminus NF^{z}_{j}; m \in M^{z}_{i_v} \tag{14}$$

for the virtual cold utility:

$$q^{z}_{im, j_v n} = 0 \qquad z \in Z; i \notin HF^{z}; m \in M^{z}_{i}; n \in N^{z}_{j_v}$$
$$q^{z}_{im, j_v n} = 0 \qquad z \in Z; i \in HF^{z}; m \in M^{z}_{i} \setminus MF^{z}_{i}; n \in N^{z}_{j_v} \tag{15}$$

Next, we define a virtual HEX technology which can be applied only to the flexible streams and virtual utilities. This can be done via the set P_t (part 2.2.2). Finally, the costs related to the virtual utilities and virtual heat exchanger technology are set to zero.

In summary, we propose a solution to consider a flexible stream whose outlet temperature is not fixed, but varies in an interval defined by the designer. The algorithm will determine the outlet temperature and the heat exchanger network which minimize the total annual cost (11).

3. Case study

We now present a series of results obtained via the developed model. The problem, originally presented in [11], consists of two hot streams, two cold streams, one hot utility and one cold utility. The streams data are shown in Table 1.

Table 1. Basis data of the problem (pinch temperature fixed to 20 K)

Stream	**T_{in} (°C)**	**T_{out} (°C)**	**Energy variation (kW)**	**h ($W.m^{-2}.K^{-1}$)**
H1	175	45	361	56
H2	125	65	667	56
C1	20	155	750	56
C2	40	112	300	56
H3 (utility)	180	179	-	56
C3 (utility)	15	25	-	56

The pinch is supposed to be equal to 20 K for all heat exchangers used. The tested scenarios are summed up in **Erreur ! Référence non valide pour un signet.**. Two HEX technologies with different costs and correlation factors will be used. While the HEX technology n°1 can be used for all streams, we keep the n°2 only for matches between streams H1 and C1. The total costs presented are results obtained by the model. In what follows, we report the heat exchanger networks obtained. Notice that splitting streams is permitted.

The test n° 1 is considered as the base test where only the HEX technology n°1 is available and there is no flexible stream. Fig. 2 shows the heat exchanger network determined by the model.

Fig. 3 reports the heat exchanger network in the test n°2 where both HEX technologies are available. One can see that this network is completely different from the one obtained from the test n°1 (Fig. 2). For the match between stream H1 and C1, the HEX technology n°2 is used because it is less expensive than the n°1. In addition, the heat exchanged is increased from 85 kW in the test n°1 to 286 kW in the test n°2, in order to get the maximum benefit from the lower cost of technology n°2. As a consequence, the total cost is reduced from 181 to 176 k$/year.

Table 2. Test conditions

Common data for all tests				
		Cost ($/year)	Correction factor	Match allowed
HEX technology *(A = HEX surface in m²)*	n°1	5292+77.8*A	1	for all matches
	n°2	4000+50*A	0.7	only for matches between streams H1 and C1
Cold utility cost	86 ($/kW/year)			

Test specifications

	Available HEX technology	Flexible stream	Hot utility cost ($/kW/year)	Total cost (k$/year)
Test 1	only n°1	no	173	181
Test 2	both n°1 and n°2	no	173	176
Test 3	both n°1 and n°2	H1 is a flexible stream with outlet temperature $\subset(45°C;65°C)$	173	174
Test 4	both n°1 and n°2	H1 is a flexible stream with outlet temperature $\subset(45°C;65°C)$	1800	598

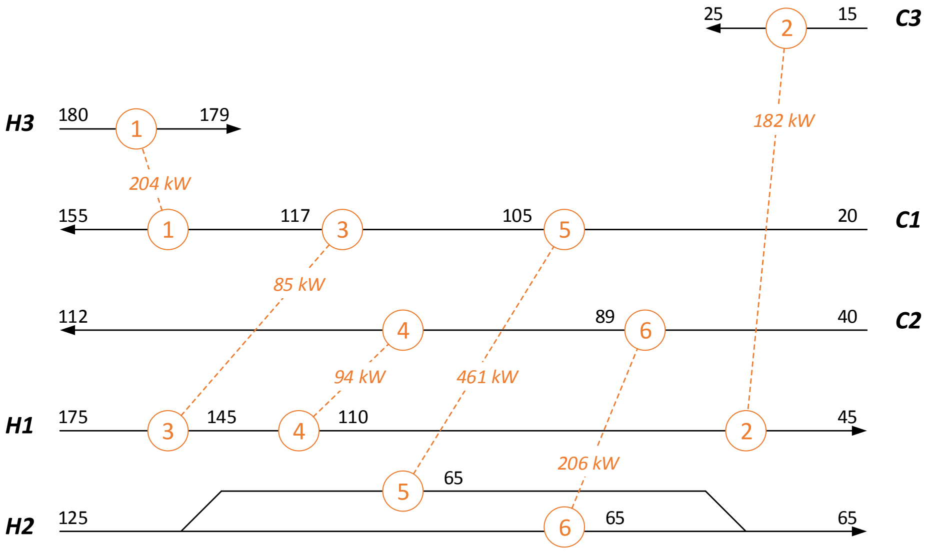

Fig. 2. Heat exchanger network (Test 1)

In the test n°3, both HEX technologies are available and we set stream H1 as a flexible stream with a outlet temperature $\subset(45°C;65°C)$. Then, stream H1 consists of two parts: a principal part H1_1 where the temperature is decreased from 175 to 65 °C, and a surplus part H1_2 in which the temperature varies from 65 to 45°C. Fig. 4 shows the heat exchanger network found by the algorithm. One can observe that the surplus part H1_2 is not used. That allows to reduce the cold utility requirement from 226 kW in the test n°2 to 211 kW in the test n°3. In addition, the total cost is decreased from 176 to 174 k$/year.

Using the surplus part of stream H1 will be interesting if the additional HEX cost can be counterbalanced by the drop of the hot utility requirement. For this reason, we carry out the test n°4 in which the hot utility cost is set to 1800 $/kW/year. Notice that this value is very high in comparison with the hot utility costs in the real life. Indeed, it is only used to show the relevance of the flexible stream functionality proposed by our model. Fig. 5 shows the heat exchanger network for the test n°4. Hence, the total heat available on stream H1 is exploited, allowing to reduce the hot utility requirement from 288 to 254 kW in the tests n°3 and n°4, respectively.

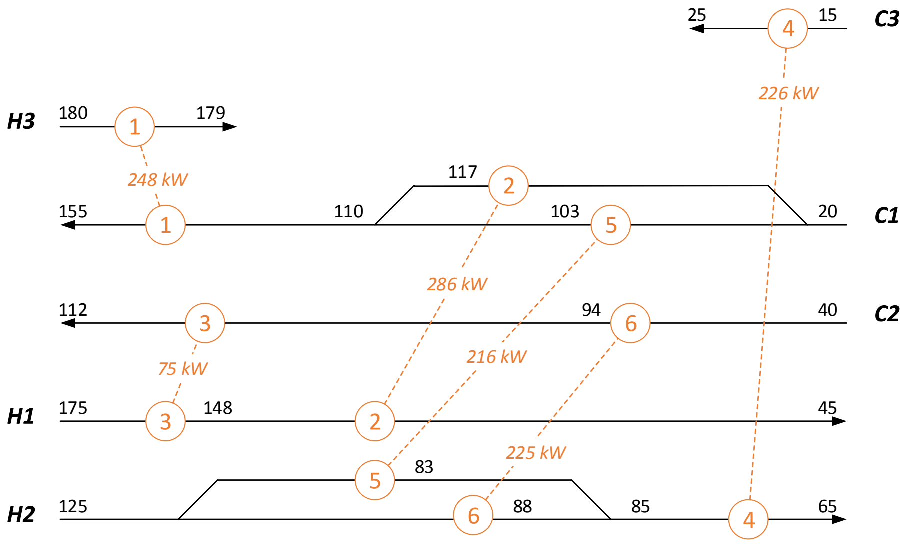

Fig. 3. Heat exchanger network (Test 2)

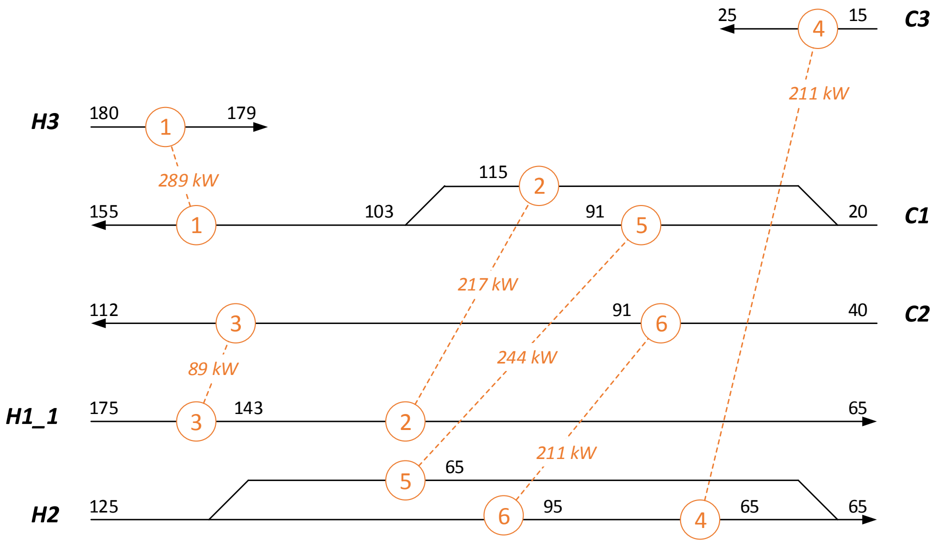

Fig. 4. Heat exchanger network (Test 3)

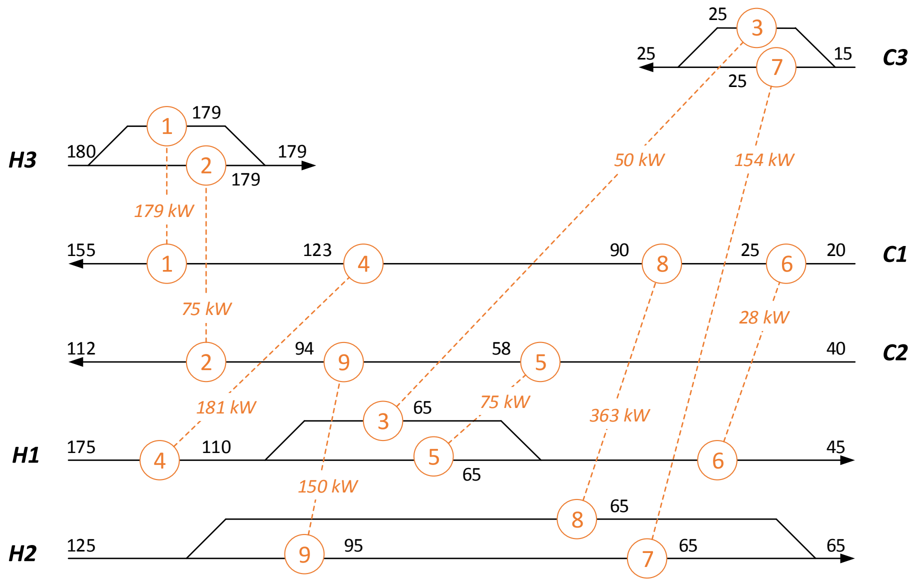

Fig. 5. Heat exchanger network (Test 4)

4. Conclusion

A new MILP model for HENS was described in the paper. The model is an extension of the one presented in [5]. While keeping all features of the original model, we introduce two new functionalities: multiple HEX technologies and flexible stream. The multiple HEX technologies functionality allows the designer to set a number of HEX technologies with different costs and performances. The second functionality considers flexible streams of which outlet temperature can vary in a range set by the designer. The algorithm can find the optimum HEX technologies and outlet temperatures which minimize the total annual cost function. To validate the relevance of these functionalities, the model was checked through a series of tests, based on a problem found in the literature. According to the obtained results, it can be concluded that the new functionalities were successfully implemented in the model. Compared to the existing models, this work contributes to designing a more realistic heat exchanger network and to proposing more design flexibilities.

Acknowledgments

This study was supported by grants of “L'Agence nationale de la recherche (ANR), France”, via CERES-2 project

Nomenclature

Sets

CF^z	{j|j is a flexible cold stream present in zone *z*}
C_n^z	{*j*|*j* is a cold stream present in temperature interval *n* in zone *z*}
CU^z	{*j*|*j* is a cooling utility present in zone *z*}($CU^z \subset C^z$)
C^z	{*j*|*j* is a cold stream present in zone *z*}
HF^z	{i|*i* is a flexible hot stream present in zone *z*}
H_m^z	{*i*|*i* is a hot stream present in temperature interval *m* in zone *z*}
HU^z	{*i*|*i* is a heating utility present in zone *z*} ($HU^z \subset H^z$)
H^z	{*i*|*i* is a hot stream present in zone *z*}
MF_i^z	{m|m is a temperature interval belonging to zone z, in which the surplus zone of stream hot i is present} ($MF_i^z \subset M_i^z$)
M_i^z	{*m*|*m* is a temperature interval belonging to zone *z*, in which hot stream *i* is present}
M^z	*m*|*m* is a temperature interval in zone *z*}
NF_j^z	{n|n is a temperature interval belonging to zone z, in which the surplus zone of stream cold j is present} ($NF_j^z \subset N_j^z$)
NI^C	{j| non-isothermal mixing is permitted for cold stream j}
NI^H	{i| non-isothermal mixing is permitted for hot stream i}
N_j^z	{*n*|*n* is a temperature interval belonging to zone *z*, in which cold stream *j*is present}
P	{(*i*, *j*)| a heat exchange match between hot stream *i* and cold stream *j* is permitted}
P_{im}^H	{*i*| heat transfer from hot stream *i* at interval *m* to cold stream *j* is permitted}
P_{jn}^C	{*j*| heat transfer from hot stream *i* to cold stream *j* at interval *n* is permitted}
P_t	{(i,j)| a heat exchange match between hot stream i and cold stream j via technology t is

	permitted}
Z	{z|z is a heat transfer zone}

Parameters

$A^z_{ij,\max}$	maximum shell area for an exchanger matching hot stream *i* and cold stream *j* in zone *z*
$A^z_{ijt,\max}$	maximum shell area for an exchanger of technology t matching hot stream *i* and cold stream *j* in zone *z*
c^H_i	cost of hot utility *i*
c^A_{ij}	variable cost for a counter current heat exchanger matching hot stream *i* and cold stream *j*
c^F_{ij}	fixed charge cost for a counter current heat exchanger matching hot stream *i* and cold stream *j*
c^A_{ijt}	variable cost for a heat exchanger of technology t matching hot stream *i* and cold stream *j*
c^F_{ijt}	fixed charge cost for a heat exchanger of technology t matching hot stream *i* and cold stream *j*
c^C_j	cost of cold utility *j*
$\Delta H^{z,H}_{im}$	enthalpy change for hot stream *i* at interval *m* of zone *z*
$\Delta H^{z,C}_{jn}$	enthalpy change for cold stream *j* at interval *n* of zone *z*
ΔT_i	temperature range of hot stream *i*
ΔT_j	temperature range of cold stream *j*
ΔT^{ML}_{mn}	mean logarithmic temperature difference between intervals m and n
$\Delta T^{partition}_{\max}$	maximum temperature step used for partitioning the temperature range
$\Delta T_{\min}$	minimum temperature difference
$FHEX_t$	correlation factor of heat exchanger technology t
h_{im}	film heat transfer coefficient for hot stream *i* in interval *m*
h_{jn}	film heat transfer coefficient for cold stream *j* in interval *n*
T^L_m	lower temperature of interval m
T^U_m	upper temperature of interval *m*

Variables

A^z_{ij}	area for a counter current exchanger matching hot stream *i* and cold stream *j* in zone *z*
A^z_{ijt}	area for an exchanger of technology t matching hot stream *i* and cold stream *j* in zone *z*
F^H_i	flow rate of hot utility stream *i*
F^C_j	flow rate of cold utility stream j
$q^z_{im,jn}$	heat transfer from hot stream i at interval m to cold stream j at interval n in zone z
U^z_{ij}	number of shells in the counter current heat exchanger between hot stream *i* and cold

stream j in zone z

U_{ijt}^{z} number of shells in the heat exchanger of technology t between hot stream i and cold stream j in zone z

Index

i hot stream

i_v virtual hot stream

j cold stream

j_v virtual cold stream

Abbriviations

HENS heat exchanger network synthesis

HEX heat exchanger

References

[1] K. C. Furman and N. V Sahinidis, “A Critical Review and Annotated Bibliography for Heat Exchanger Network Synthesis in the 20th Century,” pp. 2335–2370, 2002.

[2] J. J. Klemeš, P. S. Varbanov, and Z. Kravanja, “Recent developments in Process Integration,” *Chem. Eng. Res. Des.*, vol. 91, no. 10, pp. 2037–2053, Oct. 2013.

[3] S. A. Papoulias and I. E. Grossmann, “A structural optimization approach in process synthesis—II : Heat recovery networks,” *Comput. Chem. Eng.*, vol. 7, no. 6, pp. 707–721, Jan. 1983.

[4] J. M. Jezowski, H. K. Shethna, and F. J. L. Castillo, “Area Target for Heat Exchanger Networks Using Linear Programming,” pp. 1723–1730, 2003.

[5] A. Barbaro and M. J. Bagajewicz, “New rigorous one-step MILP formulation for heat exchanger network synthesis,” *Comput. Chem. Eng.*, vol. 29, no. 9, pp. 1945–1976, Aug. 2005.

[6] N. Pouransari and F. Maréchal, “Heat exchanger network design of large-scale industrial site with layout inspired constraints,” *Comput. Chem. Eng.*, vol. 71, pp. 426–445, Dec. 2014.

[7] K. C. Furman and N. V. Sahinidis, “Computational complexity of heat exchanger network synthesis,” *Comput. Chem. Eng.*, vol. 25, no. 9–10, pp. 1371–1390, Sep. 2001.

[8] R. Anantharaman, I. Nastad, B. Nygreen, and T. Gundersen, “The sequential framework for heat exchanger network synthesis—The minimum number of units sub-problem,” *Comput. Chem. Eng.*, vol. 34, no. 11, pp. 1822–1830, Nov. 2010.

[9] H. Becker and F. Maréchal, “Energy integration of industrial sites with heat exchange restrictions,” *Comput. Chem. Eng.*, vol. 37, pp. 104–118, Feb. 2012.

[10] I. C. Kemp, *Pinch Analysis and Process Integration*. Elsevier, 2006, pp. 99–159.

[11] U. V. Shenoy, *Heat exchanger network synthesis: Process optimization by energy and resource analysis*. Houston: Gulf Publishing Company, 1995.