Strictly robust optimal design of decentralized energy systems

Dinah Elena Majewski[a], Philip Voll[b] and André Bardow[c]

[a-c] Chair of Technical Thermodynamics, RWTH Aachen University, Aachen, Germany,
[a] dinah.majewski@ltt.rwth-aachen.de
[b] philip.voll@ltt.rwth-aachen.de
[c] andre.bardow@ltt.rwth-aachen.de (***CA***)

Abstract:

A mixed-integer linear programming (MILP) method for the strictly robust design of distributed energy supply systems is presented. The design of energy systems is a complex task and thus best addressed by mathematical optimization. However, conventional optimization models are deterministic while relying on input data which are subject to uncertainties, such as predicted future energy demands or prices. Depending on the error of the prediction, the determined optimal solution can become suboptimal or even infeasible for the actual demand. In this paper, we employ strictly robust optimization for the design of distributed energy supply systems minimizing the total annualized cost while providing higher flexibility and security of energy supply. Starting from a deterministic MILP model, the resulting robust program retains the structure of an MILP and, thus, can be solved efficiently. However, the generated strictly robust optimal solution exhibits significant additional costs compared to the deterministic optimal solution. This is due to the implicit assumption of higher demands and energy prices for the evaluation of the system costs. To enhance the comparability, in a second step, the structure and sizing of the robust solution are fixed, while the equipment operation is re-optimized for the nominal prices and demands. The resulting additional costs for the strictly robust design are only marginally larger than those of the fully deterministic problem. Our approach is used to assess the sensitivity of the total annualized costs to the uncertainties in the demands. A multicriteria approach helps the decision maker to find an appropriate trade-off between robustness and costs.

Keywords:

Decentralized energy supply systems, Mixed-integer linear programming, Robust optimization, Security of energy supply.

1. Introduction

Decentralized energy systems (DESS) can be efficiently synthesized using mathematical methods. Optimization models determine the optimal structure, sizing of the units, and operation of an energy system [1, 2]. However, these optimization models usually rely on parameters, which are assumed to be known with certainty: e.g., energy demands, prices for gas and electricity, and efficiency of the equipment. For real-world problems, these parameters have to be predicted. Usually, *perfect foresight* is assumed for the optimization.—But what happens to the designed energy system if the prediction of these parameters fails due to unexpected weather conditions or changes in the considered processes? In this case, the previously optimal solution can become suboptimal or even infeasible. For example, if the energy demand of a consumer is higher than assumed in the synthesis of the DESS, a lack of production capacity can arise. To ensure security of energy supply, *robust optimization* methods (for a review see [3]) are applied.

Robust optimization considers every possible outcome. Thereby, robust optimization avoids the need for probability distributions of the uncertain parameters, which are necessary in stochastic optimization [4]. Since these probability distributions are not known in general, robust optimization is easier applied in practice.

Strictly robust optimization was introduced for linear programs by Soyster in 1973 [5] and restated by Ben-Tal and Nemirovski [6] in 1999. Strictly robust optimization ensures the feasibility of the solution for all scenarios while minimizing the maximal costs possible considering all scenarios.

The resulting strictly robust solutions are very conservative and, therefore, expensive in general. For this reason, the frequently applied Γ-*robustness* was introduced by Bertsimas and Sim [7]. Herein, it is assumed that not all uncertain parameters vary at the same time. The level of uncertainty Γ limits the number of parameters varied simultaneously in one constraint and represents the degree of conservatism. For linear problems, the resulting program itself can be transformed into a linear program. Robust optimization approaches have been successfully applied to energy systems. Dong et al. [8] use the Γ-robustness to derive a fuzzy radial linear programming model for planning robust energy management systems with environmental constraints. Akbari et al. [9] employ Γ-robustness to optimize the installed capacity and operation of different components.

Yokoyama et al. [10, 11] use the *minimax regret* approach [12], which minimizes the maximal possible regret considering all scenarios. The regret for a scenario is the deviation between the costs, if the scenario is not known a priori, and the best possible costs. Yokoyama et al. introduce a penalty term in the objective function for unmet energy demands and propose a solution algorithm for the resulting multi-level problem, which cannot be reformulated as a linear program. A modified minimax regret approach is applied by Dong et al. [13] to determine power generation and capacity expansion for uncertain demands. The *adjustable robustness* approach [14] determines the optimal values of the variables in two steps: At first, a subset of variables is fixed by robust optimization. Only this first subset of variables has influence on the objective function. In a second step, when the occurring scenario is known, the remaining variables are determined. Street et al. [15] state a tri-level program to optimize the reserve scheduling in electricity markets with respect to possibly failing components in the energy system.

In this paper, we apply the classical strictly robust optimization by Soyster et al. [5], as described above, to the synthesis of a decentralized energy supply system. The deterministic model has been formulated by Voll et al. [1]. This MILP considers the hierarchical order of structure, unit sizing, and operation in order to design an optimal energy supply system. In this work, we consider both the energy prices and the energy demands as uncertain. For the resulting model, the concept of strict robustness has multiple advantages: First of all, it is possible to reformulate the deduced strictly robust problem also into a MILP, so it can be efficiently solved by common available solvers. Therefore, the concept of strict robustness can easily be applied. The most important feature is that security of energy supply can be guaranteed with this method, because *every* possible outcome is taken into account. Furthermore, we show that the computed strictly robust optimal design costs are only slightly more than the solution of the deterministic model.

The following section 2 gives a brief introduction to strict robustness and describes the implementation of strictly robust optimization for decentralized energy systems. In section 3, the approach is applied to a real-word problem and sensitivity investigations are performed. The results are summarized and discussed in section 4.

2. Strictly robust design framework for decentralized energy supply systems

In this section, a brief theoretical introduction to strict robustness is given, before the concept is applied to decentralized energy supply systems (DESS).

2.1. Strictly robust mixed-integer linear programming

Linear optimization problems with both continuous and integer variables are called mixed-integer linear problems (MILP):

Definition 1. Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{m\times l}$ be matrices and $c \in \mathbb{R}^n$, $d \in \mathbb{R}^l$ and $b \in \mathbb{R}^m$ be three vectors. A *mixed-integer linear problem* is given by [16]:

$$
\begin{aligned}
(\mathcal{MILP}) \quad \min \quad & c^t x + d^t y \\
\text{s.t.} \quad & Ax + By \leq b \\
& x \in \mathbb{R}^n \\
& y \in \mathbb{Z}^l.
\end{aligned}
$$

Here x and y represent continuous and integer variables, respectively. In the following section, binary variables are used to express the existence of the energy conversion units and continuous variables are used to express the sizing of the units. The vectors c and d in the objective function describe e.g. tariffs, if costs are minimized. The bound b may be interpreted as a certain budget or also as demands which have to be fulfilled at least. As mentioned before, these and further parameters may vary and, therefore, are not known with perfect foresight. The uncertain parameters are contained in the *uncertainty set* $\mathcal{U}$. In Definition 1 the deterministic problem is stated. Definition 2 introduces the *uncertain problem*. The uncertain problem contains all problems which arise from considering every single scenario one after another.

Definition 2. Let

$$\mathcal{U} := \left\{ \{\tilde{c}, \tilde{d}, \tilde{A}, \tilde{B}, \tilde{b}\} \middle| \tilde{c} \in \mathcal{U}_c \subseteq \mathbb{R}^n, \tilde{d} \in \mathcal{U}_d \subseteq \mathbb{R}^l, \tilde{A} \in \mathcal{U}_A \subseteq \mathbb{R}^{m \times n}, \tilde{B} \in \mathcal{U}_B \subseteq \mathbb{R}^{m \times l}, \tilde{b} \in \mathcal{U}_b \subseteq \mathbb{R}^m \right\}$$

be an *uncertainty set*, where $\mathcal{U}_r$ contains all possibly occurring values of parameter r with $r \in \{\tilde{c}, \tilde{d}, \tilde{A}, \tilde{B}, \tilde{b}\}$. Then $\{\tilde{c}, \tilde{d}, \tilde{A}, \tilde{B}, \tilde{b}\} =: \xi$ is called a *scenario*. The *uncertain mixed-integer linear optimization problem* $(\mathcal{MILP}_{\mathcal{U}})$ is defined by the family

$$(\mathcal{MILP}_{\mathcal{U}}) := \left((\mathcal{MILP}_{\xi}), \xi \in \mathcal{U} \right)$$

of the following problems:

$$
\begin{aligned}
(\mathcal{MILP}_{\xi}) \quad \min \quad & \tilde{c}^t x + \tilde{d}^t y \\
\text{s.t.} \quad & \tilde{A}x + \tilde{B}y \leq \tilde{b} \\
& x \in \mathbb{R}^n \\
& y \in \mathbb{Z}^l.
\end{aligned}
$$

$(\mathcal{MILP}_{\xi})$ is the optimization problem resulting for one particular scenario ξ. Thus, for each uncertain parameter, a certain value is assumed. The problem $(\mathcal{MILP}_{\mathcal{U}})$ contains all sub-problems $(\mathcal{MILP}_{\xi})$ defined by the uncertainty set. Therefore, $(\mathcal{MILP}_{\mathcal{U}})$ represents the union over all scenarios, i.e., all problems represented by all possible values for the uncertain parameters.

The *robust counterpart* minimizes the maximal possible objective function value while the constraints of the original Problem are forced to hold for every scenario.

Definition 3. The *robust counterpart* of the uncertain problem $(\mathcal{MILP}_{\mathcal{U}})$ is given by

$$
\begin{aligned}
(\mathcal{MILP}_{\mathcal{RC}}) \quad \min \quad & \sup_{\xi \in \mathcal{U}} \tilde{c}^t x + \tilde{d}^t y \\
\text{s.t.} \quad & \tilde{A}x + \tilde{B}y \leq \tilde{b} \qquad \forall \xi \in \mathcal{U} \\
& x \in \mathbb{R}^n \\
& y \in \mathbb{Z}^l,
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
\min \quad & \tau \\
\text{s.t.} \quad & \tilde{c}^t x + \tilde{d}^t y \leq \tau \quad \forall \xi \in \mathcal{U} \\
& \tilde{A}x + \tilde{B}y \leq \tilde{b} \qquad \forall \xi \in \mathcal{U} \\
& x \in \mathbb{R}^n \\
& y \in \mathbb{Z}^l.
\end{aligned}
$$

An optimal solution of $(\mathcal{MILP}_{\mathcal{RC}})$ is called strictly robust optimal for the uncertain problem $(\mathcal{MILP}_{\mathcal{U}})$. For corresponding definitions for linear problems, see [17].

The equivalence in Definition 3 holds, because we minimize the auxiliary variable τ which is an upper bound for the original target function $\tilde{c}^t x + \tilde{d}^t y$—no matter which scenario occurs. In the robust counterpart $(\mathcal{MILP}_{\mathcal{RC}})$ the constraints are satisfied for each scenario $\xi \in \mathcal{U}$, which ensures the feasibility of the solution for the whole range of uncertainties. The supremum determines the maximal possible costs over all scenarios. Thus, the resulting solution is feasible for all scenarios and minimizes the costs in the worst case scenario[1].

In this paper, we consider *interval-based uncertainties*, i.e., every entry of an uncertain vector $\tilde{r} \in \mathbb{R}^q$ can attain any value of a given interval

$$\tilde{r} \in \begin{pmatrix} [\hat{r}_1 - \varepsilon_1^r, \hat{r}_1 + \varepsilon_1^r] \\ [\hat{r}_2 - \varepsilon_2^r, \hat{r}_2 + \varepsilon_2^r] \\ \vdots \\ [\hat{r}_q - \varepsilon_q^r, \hat{r}_q + \varepsilon_q^r] \end{pmatrix}$$

without following any probability distribution. For matrices, these intervals are defined for each entry.

2.2. The strictly robust problem formulation for decentralized energy supply systems

Before applying strictly robust optimization, we present the considered deterministic optimization problem introduced by Voll et al. [1] based on [2].

2.2.1. The deterministic optimization problem with perfect foresight

Voll et al. [1] use an MILP to describe the DESS synthesis problem. The synthesis considers the optimization of the structure, sizing, and operation of the components. The existence of a component k is represented by the binary variable y_k, which is enforced by the on-off-status binary variable δ_{kt} of unit k in time-step $t \in L$. Continuous variables express the nominal size $\dot{V}_k^N$ and the provided energy-flow $\dot{V}_{kt}$ of the component k in a certain time-period Δt_t. The model comprises boilers B, combined heat and power units CHP, absorption chillers AC, and compression chillers CC as possible components of the energy supply system. Quasi-stationary energy balances are included as constraints for every time-step $t \in L$. For more details, see [1]. To obtain an optimal structure, equipment sizing, and operation, we minimize the total annualized costs:

$$\min \;\; 8760\,\mathrm{h} \cdot \left[\sum_{t \in L} \Delta t_t \left(p^{gas} \cdot \sum_{k \in B \cup CHP} \dot{U}_{kt}\left(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}\right) + p^{el,buy} \cdot \dot{U}_t^{el,buy} - p^{el,sell} \cdot \dot{V}_t^{el,sell} \right) \right] + \sum_{k \in L} \left(\frac{1}{PVF} + p_k^m \right) \cdot I_k\left(y_k, \dot{V}_k^N\right),$$

where p^{gas}, $p^{el,buy}$ and $p^{el,sell}$ denote the energy tariffs. $\dot{U}_{kt}\left(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}\right)$ is the input energy-flow in time-step t required by component k. Here, a linear approximation is used to reflect part-load performance. The output energy-flow of a unit k is denoted by $\dot{V}_{kt}$ and the purchased and sold, electricity-flow is denoted by $\dot{U}_t^{el,buy}$ and $\dot{V}_t^{el,sell}$, respectivly. The present value factor PVF is used to annualize the investment costs [18]. The factor p_k^m expresses the maintenance costs of unit k as share of the investment costs $I_k\left(y_k, \dot{V}_k^N\right)$. The investment costs are also linearized. We denote the resulting problem by $(\mathcal{MILP}^{\mathcal{DESS}})$.

---

[1] A worst case scenario is not given in general, because the supremum can also depend on more than one scenario. If there is no worst case scenario existing, the supremum of the costs over all scenarios is minimized.

2.2.2. The strictly robust optimization problem

In a DESS synthesis problem, many parameters are uncertain, such as energy prices, equipment performance curves, or demand time series. In the following, we confine ourselves to uncertainties in the gas and electricity tariffs and in all energy demands. Heating, cooling, and electricity demands in time-step t are declared by $\dot{E}_t^{heat}$, $\dot{E}_t^{cool}$, and $\dot{E}_t^{el}$. Let $\hat{\xi} := \left\{\hat{p}^{gas},\, \hat{p}^{el,buy},\, \hat{p}^{el,sell},\, \hat{\dot{E}}_t^{heat},\, \hat{\dot{E}}_t^{cool},\, \hat{\dot{E}}_t^{el}\right\}$ define the nominal scenario, which corresponds to the parameters employed in the deterministic problem with perfect foresight.

The interval-based uncertainty set is defined by:

$$
\begin{aligned}
\mathcal{U} := \Big\{ \Big\{ \tilde{p}^{gas}, \tilde{p}^{el,buy}, \tilde{p}^{el,sell}, \tilde{\dot{E}}_t^{heat}, \tilde{\dot{E}}_t^{cool}, \tilde{\dot{E}}_t^{el} \Big\} \;\Big|\; & \\
\tilde{p}^{gas} &= \hat{p}^{gas}(1+pg),\; pg \in [\min\{-1, -\varepsilon^{pg}\}, \varepsilon^{pg}]; \\
\tilde{p}^{el,buy} &= \hat{p}^{el,buy}(1+pe), \\
\tilde{p}^{el,sell} &= \hat{p}^{el,sell}(1+pe),\; pe \in [\min\{-1, -\varepsilon^{pe}\}, \varepsilon^{pe}]; \\
\tilde{\dot{E}}_t^{heat} &\in \Big[\min\Big\{0, \hat{\dot{E}}_t^{heat} - \varepsilon_t^{\dot{E}h}\Big\}, \hat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h}\Big], \\
\tilde{\dot{E}}_t^{cool} &\in \Big[\min\Big\{0, \hat{\dot{E}}_t^{cool} - \varepsilon_t^{\dot{E}c}\Big\}, \hat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c}\Big], \\
\tilde{\dot{E}}_t^{el} &\in \Big[\min\Big\{0, \hat{\dot{E}}_t^{el} - \varepsilon_t^{\dot{E}e}\Big\}, \hat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e}\Big], t \in \mathrm{L}\Big\}.
\end{aligned}
$$

Herein, the uncertainty of each parameter is expressed by the upper and lower bounds of the variation around the nominal value. Additionally, all parameters are restricted to positive values. The energy balances in problem $(\mathcal{MILP}^{\mathcal{DESS}})$ cannot be satisfied for every possible scenario ξ of the uncertainty set $\mathcal{U}$ at the same time. As a result, the strictly robust solution space is empty. Mathematically, this is due to the equality constraints representing the energy balances, which cannot be satisfied for two or more different levels of demand at the same time. Thus, the equality constraints are relaxed to inequality constraints, such that every energy demand has to be fulfilled, while overproduction is allowed. Thereby, we ensure that the desired demand is at least provided.

To obtain the strictly robust solution, we have to minimize the worst possible objective value over all scenarios. For this purpose, we complement the objective function with the supremum over all scenarios (cp. section 2.1). The resulting problem contains an infinite number of constraints—as the uncertainty set contains an infinite number of scenarios—and is thus not solvable. In the following, we transform the strictly robust counterpart of problem $(\mathcal{MILP}^{\mathcal{DESS}})$ into a MILP:

First, the relaxed energy balances are reformulated. The uncertain values $\tilde{\dot{E}}_t^{heat}$, $\tilde{\dot{E}}_t^{cool}$, and $\tilde{\dot{E}}_t^{el}$ can be replaced by their upper bounds, because the upper bounds of the uncertain intervals of the demands are larger than any value within these positive intervals. Thereby, redundant constraints are eliminated and the resulting constraints are given by:

$$
\begin{aligned}
&\sum_{k\in B\cup CHP} \dot{V}_{kt} - \sum_{k\in AC} \dot{U}_{kt}\big(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}\big) \geq \hat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h} && \forall t \in L \\
&\sum_{k\in AC\cup CC} \dot{V}_{kt} \geq \hat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c} && \forall t \in L \\
&\sum_{k\in CHP} \frac{\eta_{kt}^{el}}{\eta_{kt}^{th}} \dot{V}_{kt} - \sum_{k\in CC} \dot{U}_{kt}\big(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}\big) + \dot{U}_t^{el,buy} - \dot{V}_t^{el,sell} \geq \hat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e} && \forall t \in L,
\end{aligned}
$$

with $\dot{V}$ defining output and $\dot{U}$ input energy. The labels η_{kt}^{el} and η_{kt}^{th} denote the electrical and thermal efficiency of unit k, respectively. The upper bounds represent the worst case scenario for the demands.

Next, the strictly robust objective function needs to be adapted. For this purpose, we introduce the auxiliary variable τ, which is minimized while it limits the total annualized costs for every scenario (see section 2.1). Subsequently, the upper bound for the gas price is inserted (see (1)). This is allowed, because the highest price corresponds to the highest cost since the price is multiplied by positive numbers only: 8760 h, Δt_t, and $\dot{U}_{kt}(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt})$ are positive for all components k in every time-step t. The uncertain electricity tariffs for purchasing and selling energy vary in the same way, because the price levels are correlated. As electricity sells reduce the total annualized costs, not only the upper bound but also the lower bound of the electricity tariff has to be taken into account. Herein, the supremum (see Definition 3) depends on two scenarios and no worst case scenario exists.

As a result, the strictly robust objective function can be replaced by:

$$
\begin{aligned}
&\min \ \tau \\
&\text{s.t.} \quad 8760\,\mathrm{h} \cdot \left[\sum_{t \in L} \Delta t_t \left(\hat{p}^{gas}(1+\varepsilon^{pg}) \cdot \sum_{k \in B \cup CHP} \dot{U}_{kt}(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}) \right.\right. \\
&\qquad\qquad \left.\left. + \hat{p}^{el,buy}(1 \pm \varepsilon^{pe}) \cdot \dot{U}_t^{el,buy} - \hat{p}^{el,sell}(1 \pm \varepsilon^{pe}) \cdot \dot{V}_t^{el,sell} \right) \right] \\
&\qquad\qquad + \sum_{k \in L} \left(\frac{1}{PVF} + p^m \right) \cdot I_k(y_k, \dot{V}_k^N) \le \tau \\
&\qquad \tau \in \mathbb{R},
\end{aligned}
\tag{1}
$$

where, "$\pm$" indicates that the constraint must be satisfied for plus and for minus separately, which thus involves two different constraints. The resulting strictly robust MILP-formulation is denoted by $(\mathcal{MILP}_{\mathcal{RC}}^{\mathcal{DESS}})$ in the following.

3. Case study for strictly robust synthesis of DESS

The deduced model is applied to a real-world problem described in section 3.1. In section 3.2, the solution with perfect foresight is generated as reference for the strictly robust solution computed in the section 3.3. All solutions are identified by an automated superstructure generating algorithm, described in [1], implemented in GAMS. The optimality gap is set to 0.0 % for all calculations. The optimality gap describes the maximal relative deviation of the currently found optimal objective function value from the theoretical optimum. Finally, a sensitivity analysis is presented for the total annualized costs as function of the uncertainties of the energy demand.

3.1. The real-word Problem

A chemical park comprising six different building complexes is considered. Each building complex has time-varying heating and cooling demands [19] imposed with uncertainties (see Fig. 1). Thus, the uncertain heating and cooling demands introduced in section 2 apply for each building complex. The sum of all uncertainties of all buildings corresponds to 8.2 % of the total annual cooling demand of 27 GWh/a and 7.3 % of the total annual heating demand of 28 GWh/a. The total electricity demand is 47.5 GWh/a with 32.7 % of variation. The time-steps represent aggregated months with similar load profiles. Additionally, winter and summer peaks are considered. The load profiles and their uncertainties are deduced from real-data of previous years. The site has already two boilers, one CHP engine, one absorption chiller, and one compression chiller installed. Furthermore, the site is divided in two areas (Site A and Site B). Due to the existing infrastructure, the cooling systems of the areas cannot be connected. Gas and electricity can be purchased from the public grids at a price of 6 ct/kWh and 16 ct/kWh, respectively. The power can be fed in at a tariff of 10 ct/kWh. As mentioned above, we consider the different energy demands to be uncertain as well as the energy prices and tariffs. The uncertainties of the gas and electricity tariffs correspond to 40 % and 46 % of the original values, respectively. The cash flow time is assumed to be ten years.

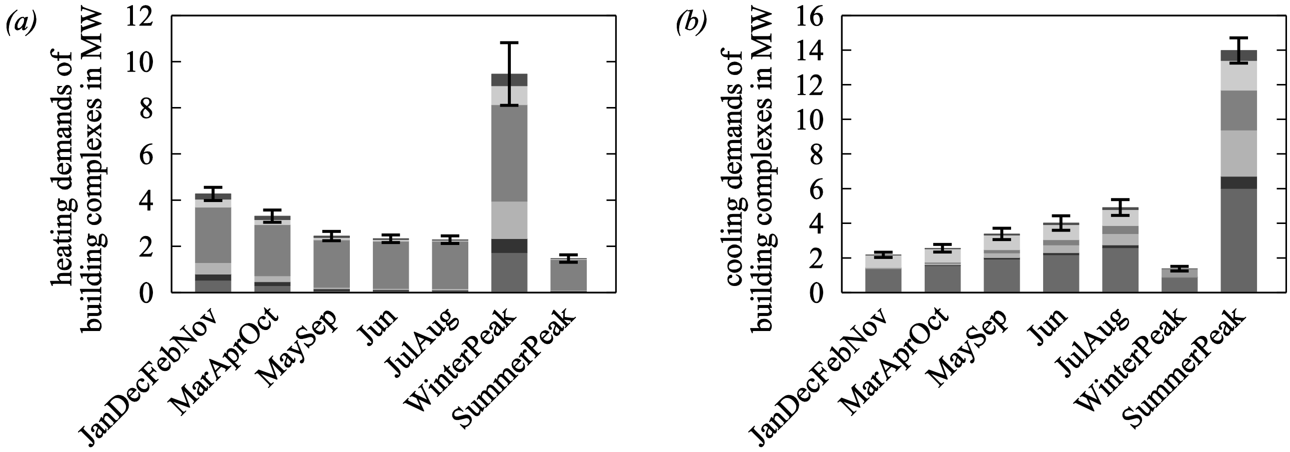

Fig. 1. Stacked bar chart of heating (a) and cooling (b) demands of building complexes for aggregated months and the summer and winter peaks. The sum of all uncertainties of the building complexes is shown as an error bar for each load period (i.e. time-step).

3.2. Optimal solution with perfect foresight

If perfect foresight is considered and thus the input-data is assumed to be known exactly, the optimal total annualized costs are 5.9 Mio.€. The corresponding solution requires the installation of two CHP engines, two absorption chillers, and two compression chillers, whereas, only one boiler and one compression chiller are retained from the old structure (see Fig. 2).

If the energy prices rise, the solution will become sub-optimal. Even worse, the solution will be infeasible, if the demands increase to their upper bounds of the considered uncertainties (see Fig. 1): The energy demands cannot be covered. To ensure security of energy supply, we apply strictly robust optimization to our model.

3.3. Strictly robust Optimization

The strictly robust problem $\left(\mathcal{MILP}_{\mathcal{RC}}^{\mathfrak{DESS}}\right)$ determines a strictly robust optimal DESS. Because the problem is a MILP itself (see section 2.2.2) it can be efficiently solved by common available optimization-solvers. The strictly robust optimal total annualized costs are 10.3 Mio.€—an increase of 74.9 % to the nominal optimal value computed with perfect foresight (see section 3.2). However, it is necessary to have in mind that the strictly robust optimal solution considers the worst case scenario for the demands, i.e., the calculated solution supplies significantly more energy than the solution of the nominal problem. Furthermore, the strictly robust problem $\left(\mathcal{MILP}_{\mathcal{RC}}^{\mathfrak{DESS}}\right)$ assumes higher prices for the energy purchased.

In order to draw a valid comparison between the nominal and the strictly robust optimal solution, the operation is re-optimized for the nominal scenario $\hat{\xi}$ while keeping the strictly robust structure and sizing. The relaxation of the energy balances (see section 2.2.2) is removed and the energy balances are again modeled by equality constraints to prevent overproduction. The resulting problem for the operation optimization is formulated as:

$$
\begin{aligned}
\left(\mathcal{MILP}_{\mathcal{RC}}^{\mathfrak{DESS}}(\hat{\xi})\right) \quad & \min \; f\left(\left(\underline{\delta_{kt}}, \underline{\dot{V}_k^N}, \dot{V}'_{kt}\right), \hat{\xi}\right) \\
\text{s.t.} \quad & \left(\underline{\delta_{kt}}, \underline{\dot{V}_k^N}, \dot{V}'_{kt}\right) \in \underset{(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}) \in \mathbb{X}}{\operatorname{argmin}} \; \sup_{\xi \in \mathcal{U}} f\left(\left(\delta_{kt}, \dot{V}_k^N, \dot{V}_{kt}\right), \xi\right) \\
& \dot{V}'_{kt} \in \mathbb{X}_{\left[\underline{\delta_{kt}}, \underline{\dot{V}_k^N}\right]}(\hat{\xi}),
\end{aligned}
$$

where f represents the nominal objective function with undetermined operation variables $\dot{V}'_{kt}$, and $\mathbb{X}$ represents the feasible region of the strictly robust problem $\left(\mathcal{MILP}_{\mathcal{RC}}^{\mathfrak{DESS}}\right)$. $\mathbb{X}_{\left[\underline{\delta_{kt}}, \underline{\dot{V}_k^N}\right]}(\hat{\xi})$ denotes the solution space of the nominal problem $(\mathcal{MILP}^{\mathfrak{DESS}})$, but with fixed variables $\underline{\delta_{kt}}$, determining the strictly robust structure, and $\underline{\dot{V}_k^N}$, specifying the corresponding strictly robust unit sizing. Generally, the feasibility of the new problem cannot be guaranteed, because the minimal part-load performance may enforce overproduction, which is prohibited by the re-enforced equality constraints. However, the problem of the case study is solvable and yields a new objective function value of 6 Mio.€. Thus, for the same conditions, the robust structure and sizing costs only 0.8 % more than the nominal optimal design. Both design-options are compared in Fig. 2. The structure is also shown in Fig. 4 in the following section. The nominal as well as the strictly robust solution both keep only one boiler and compression chiller of the already existing components on Site A, while another existing compression chiller, a boiler, and a CHP engine are not used anymore. In the strictly robust structure, the newly installed CHP engines CHP1 and CHP2 are larger than those of the nominal solution in order to fulfill increased heating demands. To cover the cooling demands, the strictly robust solution installs one additional absorption chiller AC2 on Site A and AC1 is significantly enlarged, such that a substantial smaller compression chiller CC2 is sufficient. On Site B, two additional compression chillers are installed (CC4B, CC5B), while the absorption chiller AC3B is scaled-down, compared to the nominal solution.

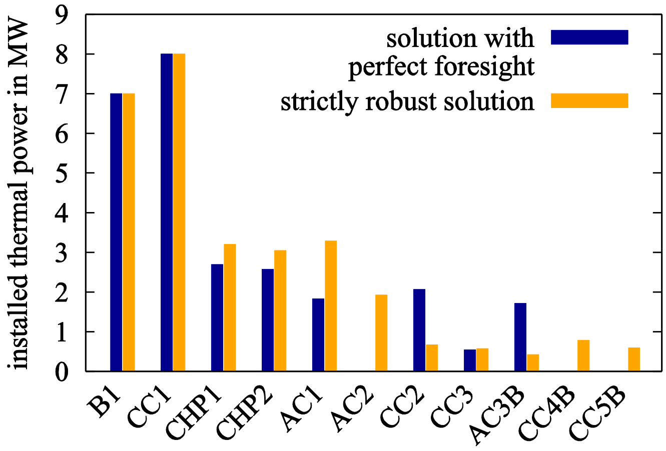

Fig. 2. Optimal design including structure and sizing of the nominal problem illustrated in dark blue; the orange bars represent the robust optimal design; only the components AC3B, CC4B, and CC5B are placed on Site B.

The corresponding total annualized costs are listed in the following Table 1.

Table 1. Total annualized costs (TAC) of the nominal problem $(\mathcal{MILP}^{\mathcal{DESS}})$, the strictly roust problem $(\mathcal{MILP}_{\mathcal{RC}}^{\mathcal{DESS}})$, and the operation-optimizing problem $\left(\mathcal{MILP}_{\mathcal{RC}}^{\mathcal{DESS}}(\hat{\xi})\right)$ for the nominal scenario with the strictly robust optimal design

problem	*TAC*, Mio.€	*Deviation from nominal TAC*, %
$(\mathcal{MILP}^{\mathcal{DESS}})$	5.91	-
$(\mathcal{MILP}_{\mathcal{RC}}^{\mathcal{DESS}})$	10.34	74.9
$\left(\mathcal{MILP}_{\mathcal{RC}}^{\mathcal{DESS}}(\hat{\xi})\right)$	5.96	0.8

Thus, a small increment of the total annualized costs can induce higher flexibility and therefore higher security of energy supply. In particular, the energy supply system features more flexibility, if the structure contains many chillers. In [20], this behavior was deduced by analyzing different near-optimal solutions of the synthesis problem. Our computed solution substantiates this conjecture: three additional chillers are installed in the strictly robust design. The reason for higher flexibility is the larger capacity of cooling. Varying heating demands can be fulfilled by shifting cooling generation either from absorption chillers to compression chillers or vice versa. Accordingly, it is sufficient to install larger heating units without adding more components.

3.4. Sensitivity analysis of the strictly robust solution

The strictly robust concept ensures that the solution is feasible no matter which scenario occurs. However, lower and upper bounds of the considered uncertain intervals are, in general, unknown themselves. Thus, we analyze the sensitivity of the total annualized costs to the variation of the demand uncertainties. In the following, we vary the lower and upper bounds of the uncertain intervals by scaling the uncertainty size $\varepsilon_t^{\dot{E}h}$, $\varepsilon_t^{\dot{E}c}$ respectively $\varepsilon_t^{\dot{E}e}$ for each time-step $t \in L$ with the parameter $\omega \in [0, 2]$. E.g., the resulting uncertain interval for the heating demand is given by

$\left[\min\left\{0, \hat{\dot{E}}_t^{heat} - \omega \cdot \varepsilon_t^{\dot{E}h}\right\}, \hat{\dot{E}}_t^{heat} + \omega \cdot \varepsilon_t^{\dot{E}h}\right]$.

Figure 3 shows the total annualized costs as function of the uncertainty sizing-factor ω.

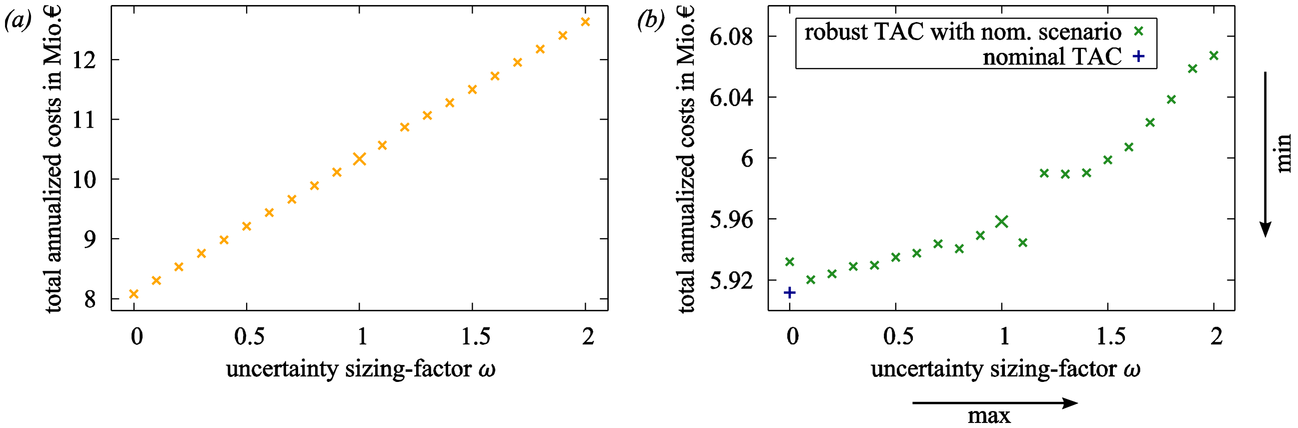

Fig. 3. Sensitivity analysis of the total annualized costs (TAC) on the uncertainty sizing-factor ω. The enlarged markers for ω =1 correspond to the results of section 3.3. In (a) the strictly robust optimal values are shown. Figure (b) depicts the total annualized costs (illustrated by green "×") with re-optimized operation relying on the strictly robust design; the blue "+" represents the optimal total annualized costs for the nominal scenario computed in section 3.2.

A higher uncertainty sizing-factor ω implicates higher robustness. As expected, increasing the sizing-factor ω leads to higher total annualized costs. Remarkable in Fig. 3 (a) is the linear trend of the curve, which shows that the costs do not increase disproportionately. The linearity is due to high changes of the operation costs (from 7.5 Mio.€ to 11.9 Mio.€) dominating the variation of the annualized investment costs (from 0.35 Mio.€ to 0.48 Mio.€). The part-load efficiencies of the components have a limited impact, such that the operation costs depend nearly linearly on the demand.

However, following the explanation of section 3.3, we re-optimize the operation of the given strictly robust optimal design for the nominal scenario to obtain a realistic comparison of the cost: Hence, we fix the variables determining the structure of the DESS and the sizing of the components. The optimal values for the operation-variables are calculated anew for the nominal scenario. Equality constraints are re-inserted to preclude overproduction. The resulting total annual costs are represented in Fig. 3 (b).

As observed in section 3.3, the cost-increase for the robust solution is linear in the nominal scenario. Noticeable are the jumps in the curve of Fig. 3 (b). These jumps exists due to different numbers of installed units: When the uncertainty sizing-factor ω changes from 0 to 0.1, one compression chiller less is needed, while another absorption chiller with lower investment costs should be installed. Changing ω from 1 to 1.1, the amount of compression chillers reduces by one. When ω is equal to 1.2 instead of 1.1, the strictly robust design includes one additional, expensive CHP engine. These variations in the structure are shown in Fig. 4.

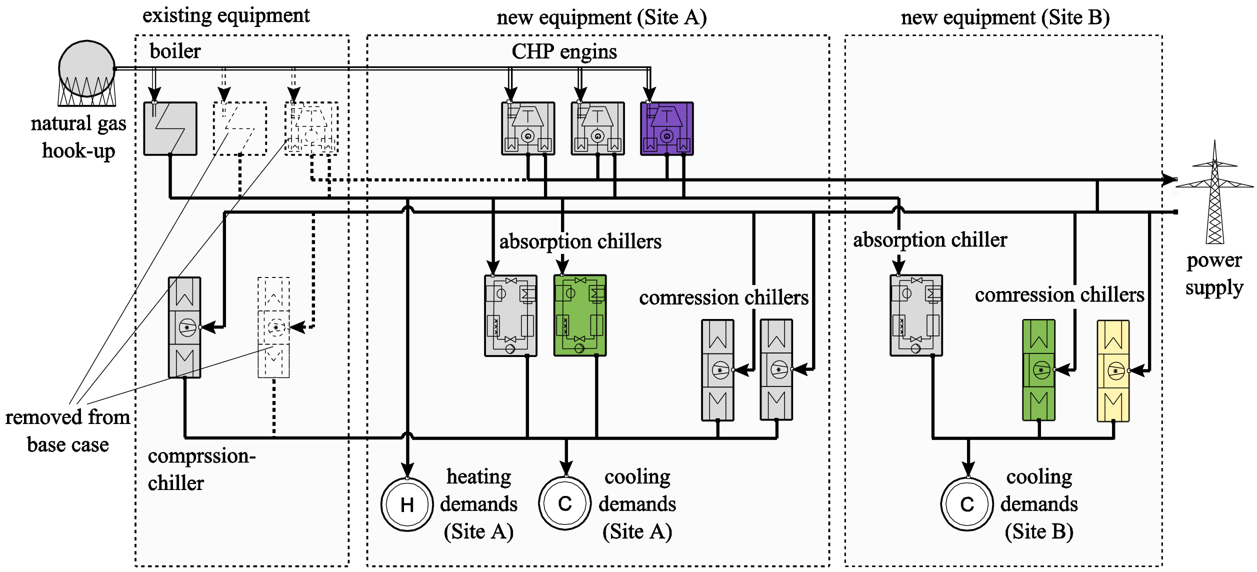

Fig. 4. The gray components illustrate the optimal structure for the deterministic model (adapted from [1]). The union of the gray, green, and light yellow components presents the robust optimal structure with uncertainty sizing-factor ω =1, the gray and green units the one with factor 1.1, and the gray, green, and dark violet with factor 1.2.

The sensitivity analysis motivates a multi-objective problem: The uncertainty sizing-factor is maximized, while the total annualized costs are minimized (see Fig. 3 (b)). On the one hand, this multi-criteria analysis favors high robustness to ensure energy supply and, on the other hand, aims at economic viability by low annual costs. These two criteria are in general contradicting and a non-trivial *Pareto-front* can be observed. For an elaboration of multi-criteria optimization see [21]. In particular, not all computed objective function values contribute to the Pareto-front: For ω equal to 0, 0.7, 0.9, 1, or 1.2, there are other solutions which are better concerning the total annualized costs as well as the sizing-factor ω (see Fig. 3 (b)). In fact, the strictly robust design computed in section 3.3 ($\omega = 1$), is not efficient, because the optimal solution generated with $\omega = 1.1$ yields a higher robustness at lower total annualized costs. Thus, the strictly robust structure and sizing generated by using $\omega = 1.1$ shold be prefered. As a result, this multi-criteria analysis identifies good trade-offs between the total annualized costs and the robustness of the solution.

4. Conclusion

We apply the concept of strict robustness to optimization of decentralized energy supply systems. A simplification for interval-based uncertainties is employed, which allows reducing the problem complexity to an MILP. Thereby, strict robustness can easily be applied to decentralized energy supply system problems. The resulting MILP models can be solved by common available solvers.

Subsequently, the presented approach is applied to a real-world case study from the pharmaceutical industry. The costs of the strictly robust optimal solution exceed the costs for the deterministic case by 75 %. In order to achieve a sound comparison, the equipment of the strictly robust design is employed for optimal operation for the nominal input parameters. The resulting costs for the strictly robust design are only slightly larger (0.8 %) than in the fully deterministic problem. The sensitivity of the total annualized costs to the size of uncertainty is analyzed in a multicriteria approach. The analysis identifies solutions with higher robustness at lower additional costs. With the presented method, energy supply systems featuring higher flexibility and security of supply at only marginally additional costs can be designed. The multicriteria evaluation can help the decision maker to find an appropriate trade-off between robustness and expected total annualized costs.

Acknowledgments

Funded by the Excellence Initiative of the German federal and state governments.

Nomenclature

AC set of absorption chillers, which might be installed

B set of boilers, which might be installed

CC set of compression chillers, which might be installed

CHP combined heat and power units, which might be installed

$\dot{E}^{heat}$ heating demand, kW

$\dot{E}^{cool}$ cooling demand, kW

$\dot{E}^{el}$ electricity demand, kW

I investment costs, €

L set of time-steps

$p^{el,buy}$ tariff for purchasing electricity, €

$p^{el,sell}$ tariff for selling electricity, €

p^{gas} tariff for purchasing gas, €

p^{m} fraction of the investment costs to calculate the maintenance costs

PVF present value factor

$\mathcal{U}$ uncertainty set

$\dot{U}$ input energy-flow, kW

$\dot{U}^{el,buy}$ purchased energy-flow, kW

$\dot{V}^{el,sell}$ sold energy-flow, kW

$\dot{V}^{N}$ installed nominal power-flow, kW

y binary variable for the existence of a unit

Greek symbols

δ on-off-status binary variable

$\vartriangle t$ duration of a time-period, a

ε^{pg}, ε^{pe} relative uncertainty for the gas and electricity tariffs

$\varepsilon^{\dot{E}h}$, $\varepsilon^{\dot{E}c}$, $\varepsilon^{\dot{E}e}$ absolute uncertainty for heating, cooling and electricity demand, kW

η^{el}, η^{th} electrical and thermal efficiency

ξ scenario

τ auxiliary variable

ω uncertainty sizing-factor

Subscripts and superscripts

k component

t time-step

ˆ nominal input value

˜ uncertain input value

_ fixed variable

References

[1] Voll P., Klaffke C., Hennen M., Bardow A., Automated superstructure-based synthesis and optimization of distributed energy supply systems. Energy 2013;50:374–388.

[2] Yokoyama R., Ose S., Optimization of energy supply systems in consideration of hierarchical relationship between design and operation. ECOS 2012: Proceedings of the 25th International

Conference on Efficiency, Cost, Optimization, Simulation, and Environmental Impact of Energy Systems; 2012 Jun 26-29; Perugia, Italy. Firenze University Press:389:1–13.

[3] Goerigk M. Algorithms and Concepts for Robust Optimization [dissertation]. Göttingen, Germany: Georg-August Universität Göttingen; 2012.

[4] Birge JR., Louveaux F., Introduction to Stochastic Programming. 2nd ed. Springer; 2011.

[5] Soyster AL., Technical Note – Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming. Operations Research 1973;21(5):1154–1157.

[6] Ben-Tal A., Nemirovski A., Robust solutions of uncertain linear programs. Operations Research Letters. 1999;25(1):1–13.

[7] Bertsimas D., Sim M., The Price of Robustness. Operations Research 2004;52(1):35–53.

[8] Dong C., Huang GH., Cai YP., Liu Y., Robust planning of energy management systems with environmental and constraint-conservative considerations under multiple uncertainties. Energy Conversion and Management 2012;65:471–486.

[9] Akbari K., Nasiri MM., Jolai F., Ghaderi SF., Optimal investment and unit sizing of distributed energy systems under uncertainty: A robust optimization approach. Energy and Buildings 2014;85:275–286.

[10] Yokoyama R., Ito K., Murata T., Robust Optimal Design in Multistage Expansion of a Gas Turbine Cogeneration Plant Under Uncertain Energy Demands. Journal of Engineering for Gas Turbines and Power 2004;126(4):823.

[11] Yokoyama R., Fujiwara K., Ohkura M., Wakui T., A revised method for robust optimal design of energy supply systems based on minimax regret criterion. Energy Conversion and Management 2014;84:196–208.

[12] Yaman H., Karasan OE., Pinar MC., The robust spanning tree problem with interval data. Operations Research Letters 2001;29:31–40.

[13] Dong C., Huang GH., Cai YP., Xu Y., An interval-parameter minimax regret programming approach for power management systems planning under uncertainty. Applied Energy 2011;88(8):2835–2845.

[14] Ben-Tal A., Goryashko A., Guslitzer E., Nemirovski A., Adjustable robust solutions of uncertain linear programs. Mathematical Programming, Ser A 2003;99:351–376.

[15] Street A., Moreira A., Arroyo JM., Energy and Reserve Scheduling Under a Joint Generation and Transmission Security Criterion: An Adjustable Robust Optimization Approach. IEEE Transactions on Power Systems 2014;29(1):3–14.

[16] Floudas CA., Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications. Oxford University Press 1995.

[17] Ben-Tal A., Ghaoui LE., Nemirovski A., Robust Optimization. Princeton and Oxford: Princeton University Press; 2009.

[18] Broverman SA., Mathematics of Investment and Credit. 5th ed. ACTX Publications, Inc. 2010.

[19] Voll P., Automated Optimization-Based Synthesis of Distributed Energy and Supply Systems [dissertation]. Aachen, Germany: RWTH Aachen University; 2013.

[20] Voll P., Hennen M., Klaffke C., Lampe M., Bardow A., Exploring the near-optimal solution space for the synthesis of distributed energy supply systems. Chemical Engineering Transactions 2013;35:277–282.

[21] Ehrgott M., Multicriteria Optimization. Auckland, New Zealand: 2nd ed. Springer; 2005.