Optimizing the design of a district heating network

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Abstract:

District heating (DH) system is one of the most important infrastructures in cold area. It is very important to balance the initial investment and the pumping cost in the DH network design. Currently, this design is mostly based on a recommended value of $R=\Delta P/L$ (specific pressure loss) in the main lines. This will result in a feasible network but probably not the optimal in most cases, which means that it is still possible to improve the design further. The aim of this paper is to develop a multi-objective optimization model to facilitate the design considering the initial investment of the pipes and the pumping cost for heat distribution. The main line in the optimization method is generating all possible network scenarios consisting of different series of diameters for each pipe in the network. Then for each DH network scenario, the annuity of initial investment, the pumping cost and the total annual cost will be calculated using different combinations of steel prices and pump operating time levels. Finally, we can obtain the optimization results and find out the overall optimal design. The optimization method is demonstrated in an example DH network and the optimal scope of R values is found from 44.9 Pa/m to 79.3 Pa/m which justify the recommended R values. But we conclude that on one hand R should be less than 100 Pa/m but on the other hand it should not be too small, otherwise the total annual cost will increase due to the large initial investment. In addition, we find that the pumping cost percentage is always increasing along with the increasing R values. This percentage increases from 41% to 82%. Therefore, pumping cost is a more important factor when designing a DH network and it should be given more considerations compared to the initial investment.

Keywords:

District Heating, Design, Optimization, Pumping Cost, Initial Investment, Network.

1. Introduction

District heating (DH) system is one of the most important infrastructures in cold area. Recently, the DH consumption and connected heat load has been increasing in many countries even though the specific DH consumption is gradually decreasing. In Finland, the number of DH customers has increased from 85,000 to 140,000 during 2000-2013 and simultaneously the total length of networks increased from less than 9,000 km to 14,000 km [1]. The situation is similar in China, e.g. the hot water and vapour DH networks have been growing very fast since the DH areas and connected DH loads are booming in recent years [2,3]. Therefore, it can be foreseen that more DH networks will be designed and constructed around the world in the future.

The dimensioning of the DH network is highly coupled with hydraulic conditions [4]; but for simplicity, in DH engineering the network design is often based on an recommended value of $R = \Delta P/L$ (specific pressure loss) in the main lines. For example in China R is recommended as 30-70 Pa/m [5] and in Finland R should be less than 1 bar/km namely 100 Pa/m. This will result in a feasible network but probably not then optimal in most cases, because DH installations and requirements may differ in different DH areas, which means that it is still possible to improve the design further. This necessitates optimizing the network design based on many factors to ensure its capacity and efficiency. Hassine and Eicker [6] described a Gemany DH network based on the graph theory and the Newton algrithm was used to solve the system of nonlinear equations. They concluded that the the different geographical distributions of consumers within the network have a slight impact on the primary energy use and on the CO_2 emissions of the system. The same optimization method was adopted by Wang et al. [7]. Tol and Svendsen [8] tried to improve the dimensioning of piping networks and network layouts in low-energy DH systems connected to low-energy buildings in Roskilde, Denmark. Pirouti et al. [9] carried out an analysis on energy consumption and economic performance of a DH network. They stated that the supply and return temperatures and operating strategies influence the annual energy consumption and the equivalent annual cost. The design case with minimum annual total energy consumption and equivalent annual cost had different pipe diameters and pump sizes under different operating strategies and temperature regimes. Koiv et al. [10] proposed a new dimensioning method of DH network for the tree figure network. The method is based on a probabilistic determination of the flow rate for hot water heating. They calculated the heat loss and the pumping cost for heat distribution. Through an example calculation with 10 consumers, they reported that the dimesioining method can decrease the power of boilers by 45%, the cost of the DH network by 12%, and the pumping cost by 35%. Ancona et al. [11] introduced a technical-economical optimization procedure for DH network design in order to both minimize the pumping energy consumption and the thermal energy losses and maximize the annual revenue.

This paper proposes a novel optimization method for the DH network design. The design is influneced by many factors, among which the initial investment of pipes and the pumping cost are two main considerations, because the initial investment will have a great impact on the construction of the network and the pumping cost represents the main operating cost after the construction. However, these two factors are conflicting objectives; if we use big pipes in the network, then the pumping cost is lower but the initial investment will increase dramatically and vice versa. This means that we cannot optimize the network trivially; instead we should employ the multi-objective optimization method to find out an optimal balance between the pipe investment and pumping cost. The aim of this paper is to develop such an optimization method to optimize the design considering the initial investment of the pipes and the pumping cost for heat distribution. The heat losses in the distribution should be considered when evaluating the energy consumption of a DH network [12]. However, in this network optimization study, the impact of heat loss is so small that can be relaxed. This is justified by a heat loss calculation in the second international DHC (district heating and cooling) summer school [13], where it was found that the heat loss accounts for only 2-3% of the total distributed heat if the insulation is in good condition.

2. Methods

In the DH network optimization study, we cannot dimension the network using the already known value of *R*, because our purpose is to determine it for a specific DH network through optimization. This paper develops a multi-objective optimization model for this purpose. The main process is as follows:

1. Calculate the mass flow rate in each pipe and the pump.

The heat load profile and the network layouts of the connected heat users (consumers) are already known before the network optimization, so that we can start the optimization by calculating the mass flow rate in each pipe and in the pump by (1),

$$G_i' = \frac{Q_{load,i}'}{c\left(\tau_1' - \tau_2'\right)}, \qquad (1)$$

where G_i' is the mass flow rate of pipe *i*, kg/s; $Q_{load,i}'$ is the heat load for pipe *i*, W; *c* is the specific heat capacity of water, *c* = 4186.8 J/(kg °C); τ_1' and τ_2' are the design supply and return water temperatures, °C.

2. Determine the possible combinations of diameters of the pipes.

This step is a little troublesome, because the diameters are discrete nominal diameters in DH engineering and we need to consider a large number of combinations if the number of the pipes is not so small. From this point of view, the problem is similar with the combinatorial optimization problem [14] that consists of finding an optimal object from a finite set of objects. However, in our optimization problem, not all combinations will be used in the optimization; instead, only the possible combinations will be selected in next steps. The possible combinations are defined by the following rules: 1) the diameters of upstream pipes are larger or equal to the consecutive downstream pipes in the supply main line, and the order is reversed in the return main line; 2) the supply and return main lines are symmetrical. It can be found that if there are tens of heat users the combination size are still reasonable that can be treated by computers although not trivially. For real-life normal tree shape DH network, it is common to have tens of pipes in the main line which extends up to more than ten kilometres. However, in case of too many heat users the problem size can be reduced by some aggregating methods used in DH simulation [15,16].

3. Calculate the water velocity, pressure loss ΔP in each pipe and specific pressure loss R.

In this step we will calculate these variables for each feasible combination of pipe diameters obtained by step 2. The water velocity can be calculated very fast if we know the pipe diameter and corresponding mass flow rate. Specific pressure loss R in each pipe can be calculated by (2) [17],

$$R = \frac{\lambda G_i^2}{\rho d_i^5}, \qquad (2)$$

where λ is the fraction factor of the inner pipe surface, G_i is the mass flow rate in each pipe, kg/s; ρ is water density, kg/m^3; d is inner diameter of pipe i, m. In general, λ is a function of Reynolds number Re and relative pipe roughness ε; where $Re = \rho v d/\mu$, μ is the dynamic viscosity coefficient of water, kg/(m s) and $\varepsilon = K/d$, K is the inner surface roughness, m. The value of λ can be obtained from the Moody diagram, but in the optimization we propose calculate it by (3) (Альтщуль equation or Altschul equation) [17],

$$\lambda = 0.11\left(\frac{K}{d} + \frac{68}{Re}\right)^{0.25}, \qquad (3)$$

In addition to the frictional pressure loss, we still need to consider the local pressure loss which is caused by other components in the network e.g. valves, joints, compensators, et al. In this study, the local pressure loss is assumed to be 30% of the frictional pressure loss according to the statistical analysis of the main lines in many Chinese DH network [17].

4. Divide pipe combination scenarios according to R.

In this step, R reflects the pipe size used in the network, i.e. for a specific mass flow rate, the bigger R values corresponds to the smaller pipes and vice versa. Therefore, dividing the pipe combinations based on R means dividing them by pipe diameters. Although we have too many feasible pipe combinations, we only have relatively small number of network scenarios consisting of different series of pipe diameters for each pipe of the network. For example, we may have network scenarios with overall large pipes, intermediate level pipes and small pipes, the pipes in each scenario can have different diameters according to the mass flow rate in them, but their R values are within the same scale. Let us see what happened to one pipe; it is possible to select more than three diameter levels for this pipe, with an R value varying from a little more than zero up to e.g. 600 Pa/m, which is so high. However, if we take into account the combination rules stated in step 2, then the number of possible diameter levels for this pipe is reduced, and this is happening to all pipes in the network. In all, we will have much less network scenarios to be optimized after this step.

5. Calculate the steel consumption in cubic meters and the initial investment for each network scenario using variable prices of steel.

After we obtain the pipe diameters (the thickness is known) in each network scenario, we can calculate the steel consumption in cubic meters very easily and then the initial investment of each network scenario can also be computed using variable prices of steel considering the construction and material cost in laying the pipes. We can use many price levels reflecting the uncertainty and changing of steel price in this optimization model.

6. Determine the lift head and equivalent full operating time of the pump using variable full operating percentage in a year.

Firstly, we should calculate the lift head of pump for each network scenario by adding the pressure loss in supply and return main lines, the available pressure in heat users, the pressure drop in heat plant and the static pressure to avoid evaporation in the network. Secondly, we want to examine the impact of operating time, so that we set up three full operating percentages i.e. 40%, 60% and 80% in a year (8760 h if not a leap year). It is possible to have more operating percentages if needed in the optimization.

7. Calculate the pump power and electric motor power.

The pump power and the electric motor power can be calculated by (4) and (5),

$$P_a = \frac{\rho g Q H}{\eta}, \qquad (4)$$

$$P_e = \frac{S}{\eta_m} P_a, \qquad (5)$$

where P_a is pump axis power, W; g is gravity constant, g = 9.81m/s^2; Q is volume flow, m^3/s; H is the lift height of the pump, m; η is the pump efficiency (0.7-0.9); P_e is the electric motor power, W; S is the assurance coefficient larger than 1; η_m is the transmission coefficient.

8. Calculate the electricity consumption and pumping cost for different network senarios.

The electricity consumption for heat distribution is calculated by (6),

$$E = P_e \cdot n_0 = 0.001 \frac{\rho g Q H S}{\eta \eta_m} n_0 \text{ (kWh)}, \qquad (6)$$

where n_0 is the full power operating time of the pump, h; 0.001 is the coefficient of W to kW. Then the pumping cost can be obtained considering the electricity price. Note that the lift height, volume flow and efficiency may vary according to the working point of the pump, but we consider all the effects in defining different operating percentages in step 6 to make the optimization easier to apply.

9. Calculate the annuity of the initial investment and total annual cost.

The idea is that we assume investment I is financed by taking a loan for n years with effective interest rate r and the loan is paid back with fixed constant annual amounts A. Then the net present value (NPV) of a payment after i years is,

$$A/(1+r)^i = (1+r)^{-i} A, \qquad (7)$$

To determine the annual payment, the NPV of payments are equal to the loan,

$$I = \sum_{i=1}^{n} (1+r)^{-i} A \Rightarrow A = \left(\frac{1}{\sum_{i=1}^{n} (1+r)^{-i}} \right) I = aI, \qquad (8)$$

Here the annuity factor a is obtained by computing the sum of the geometric series,

$$a = \frac{1}{\sum_{i=1}^{n} (1+r)^{-i}} = \frac{r}{\left[1-(1+r)^{-n}\right]}, \qquad (9)$$

Now the annuity of the initial investment A can be calculated by (8) and the total annual cost is then obtained by adding the annuity of the initial investment and the pumping cost together.

10.Find the optimization results and show it.
In the last step, we will show the optimization results and find out the optimal DH network design. To conclude, the overall flowchart of the proposed DH network optimization is shown in Fig. 1.

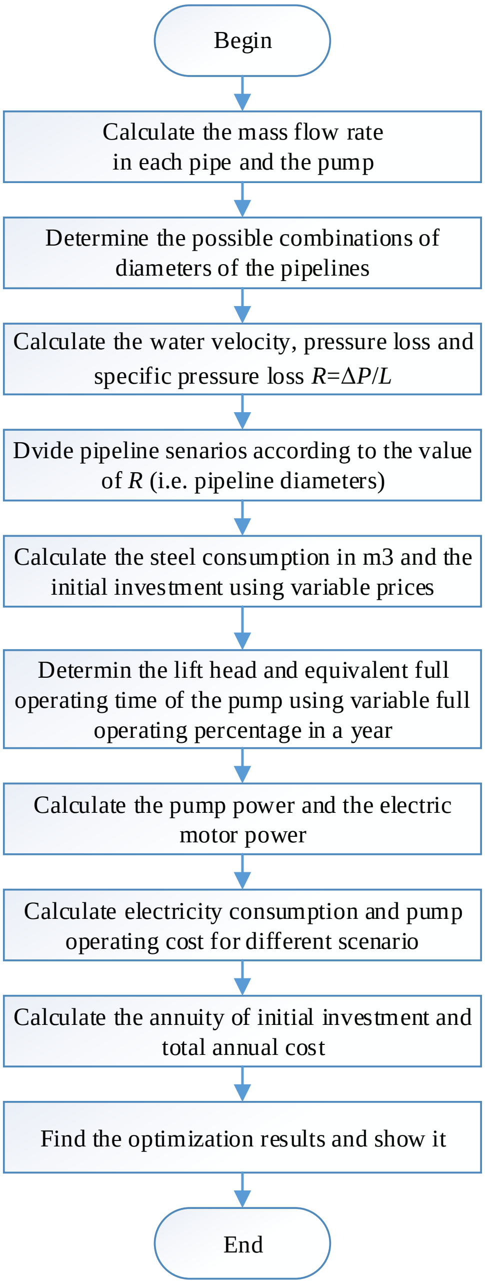


Fig. 1. The flowchart of DH network optimization.

3. Results and discussion

3.1 Example DH network

We demonstrate our optimization method through an example DH network design. The network layout is shown in Fig. 2 and length of each pipe is already known; the design supply and return water temperatures are 130/70 °C, the available pressure in each heat user (filled circle) is 50 kPa, the heat demands of all heat users are shown in Table 1. Our objective is to determine the diameters of the main line using the optimization method.

Table 1. The design heat demand profile for all heat users

Heat user NO.	1	2	3
Heat demand (GJ/h)	3.518	2.513	5.025
Heat demand (W)	977,222	698,056	1,395,833

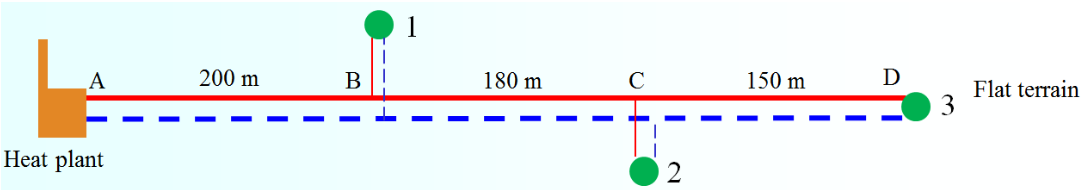


Fig. 2. The layout of a DH network, the solid and dash lines denote the supply and return pipes.

Other technical and economic parameters used in the optimization are shown in Table 2.

Table 2. Relevant parameters used in the optimization

Parameter	Description
Pressure drop in the heat plant	80 kPa
Static pressure	20 mH2O (1kPa ≈ 0.102 mH2O)
Local resistance pressure loss rate	30%
Interest rate	7.56%
Calculation period	20 years
Steel price for laying pipes	360,000 (L); 420,000 (I); 480,000 (H) Yuan/m^3 ♣
Pump operating time percentage	40%(L); 60%(I); 80%(H) of full power operating time ♣♣
Pump efficiency	70%
Water density (average)	958.4 kg/m^3
Assurance coefficient *S*	1.15
Transmission coefficient η_m	80%

♣Chinese currency RMB, 1 US dollar ≈ 6.2 Yuan recently; L= Low price scenario, I = Intermediate price scenario, and H = High price scenario, which can reflect the uncertainty and changing of steel price.

♣♣Equivalent full power operating time percentage in a year (8760 h if not a leap year), L, I and H have the same meanings as in the steel price.

3.2 Optimization results

The first three optimization steps are easy to perform for this example, but in the future we will develop more general routines to speed up the optimization process for DH network with more heat users. In step 4, we divide the network scenarios according to the *R* values shown in Table 3. It can be seen that we get 6 network scenarios and the *R* values are from 8.1 Pa/m to 568.7 Pa/m, which covers a wide range of specific pressure losses. Therefore, there is no need to extend the *R* values bigger than 568.7 Pa/m or smaller than 8.1 Pa/m. Observe that the *R* values are not integers or an arithmetic sequence, because the pipe diameters are not consecutive but discrete values and the hydraulic calculations are strongly nonlinear.

Table 3. The network scenarios for the example DH network

Scenario NO.		1	2	3	4	5	6
$R = \Delta P/L$ (Pa/m) ♣		8.1~54.7	24.3~54.7	44.9~79.3	117.5~227.2	178.3~383.5	383.5~568.7
Pipe diameters (mm) ♣♣	A->B	219×6	159×4.5	159×4.5	133×4	108×4	108×4
	B->C	133×4	133×4	133×4	108×4	108×4	89×3.5
	C->D	133×4	133×4	108×4	89×3.5	89×3.5	76×3.5

♣We use the *R* values to define the network scenarios, but these *R* values are not integers or an arithmetic sequence, because the pipe diameters are not consecutive values and the hydraulic calculations are nonlinear.

♣♣Diameters are nominal values with wall thicknesses, inner diameters are used in hydraulic calculations.

Then the following steps 5~8 calculate the initial investment of pipes and pumping cost considering different combinations of steel prices and full power operating time percentages of the pump. Subsequently in step 9, we can calculate the annuity of the initial investment by (7)-(9), and then obtain the total annual cost by adding the annuity of initial investment and the pumping cost together. For example, the results with the intermediate steel price (420,000 Yuan/m^3) and pump operating time (60% of a year) are shown in Figs. 3 and 4.

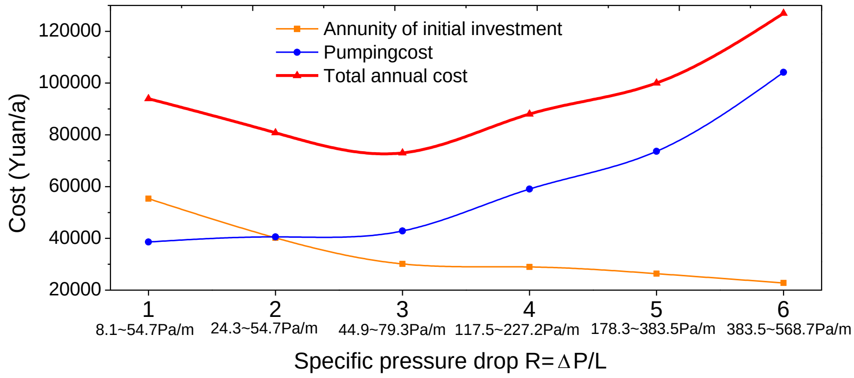
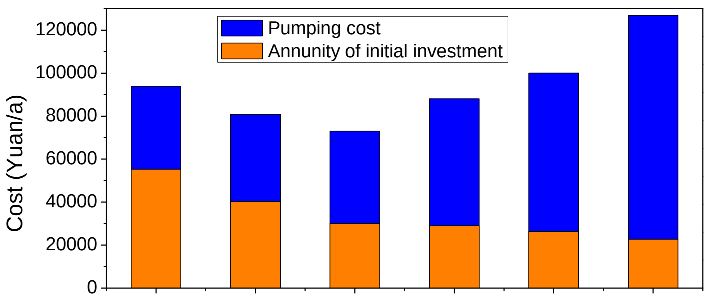
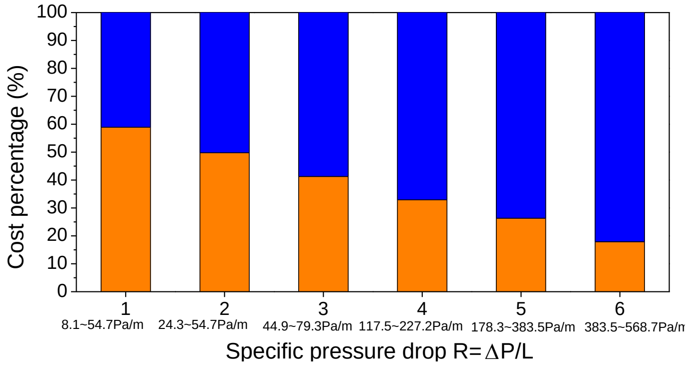


Fig. 3. The annuity of initial investment, pumping cost and total annual cost at different R values for the example DH network with the intermediate steel price and pump operating time.

Figure 3 shows the relationship of annuity of initial investment, pumping cost and total annual cost with the *R* values. Note that *R* values are not consecutive within the 6 scenarios and there may be some overlap between two successive scenarios. This is already explained in Table 3. In addition, the reason that we connect all calculation points in Fig.3 is just to show the trends of each variable. It can be found that the annuity of initial investment is a monotone decreasing function with increasing *R* values, while the pumping cost is a monotone increasing function. However, total annual cost which is the sum of the previous two variables has a minimum value when *R* is 44.9~79.3 Pa/m in scenario 3.



(a)



(b)

Fig. 4. Comparison of annuity of initial investment and pump operating cost at different R values for the example DH network with the intermediate steel price and pump operating time: a) cost value, b) cost percentage.

From Fig. 4, we can visually compare the annuity of initial investment and pumping cost. Fig. 4(a) shows the cost value of them while Fig. 4(b) illustrates the cost percentage for different network scenarios. We find that the pumping cost percentage is always increasing along with the increasing *R* values. This percentage increases from 41% to 82% within the 6 network scenarios. That is to say, if we use too thin pipes in the DH network (*R* values are large) the pumping cost will increase very fast. However, the cost percentage of annuity of initial investment is only 18%~59%, which means that the pumping cost is a more important factor when designing a DH network and it should be given more considerations compared to the initial investment.

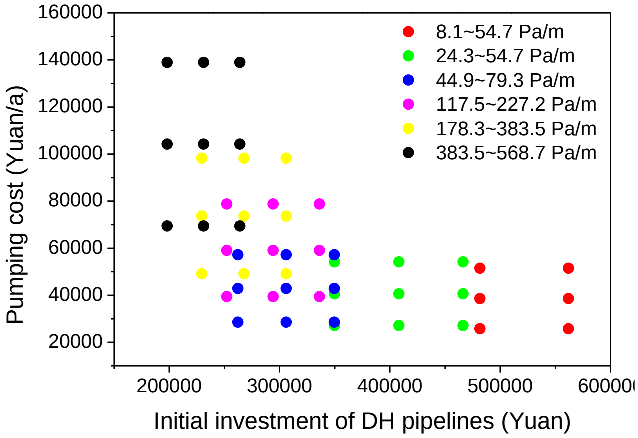


Fig. 5. Optimization results of the pumping cost and initial investment for the example DH network.

Figures 3 and 4 only indicate one possible situation in the optimization process, however Fig. 5 shows the overall optimization results for all network scenarios and all possible combinations of steel prices and pump operating time. It is unnecessary to calculate the annuity of investment in Fig. 5, therefore it is more straightforward to examine all results and find out the optimal design scenario. For this example DH network, the optimal design is the network scenario 3 with *R* values from 44.9 Pa/m to 79.3 Pa/m. Part of the optimal DH network design table is shown in Table 4.

Table 4. Example DH network design table (partly)

Pipe NO.	design flow rate G' (t/h)	length (m)	Diameter (mm)	Water velocity (m/s)	*R* (Pa/m)
	1	2	3	4	5
A->B	44	200	159×4.5	0.72	44.9
B->C	30	180	133×4	0.71	54.7
C->D	20	150	108×4	0.74	79.3

3.2 Discussion

Designing a DH network by the recommended *R* values will result in a feasible solution, but probably not the optimal in most cases. This means that it is still possible to improve the network design considering different DH parameters and configurations, because they can lead to different optimal designs. This paper proposes a novel optimization method for that purpose. The main line in the optimization method is generating all possible network scenarios consisting of different series of diameters for each pipe in the network. There are many possible diameters for one pipe disregarding the upstream and downstream pipes, but the series of diameters for one network scenario is strongly constrained by the two rules in step 2. Therefore, the number of network scenarios is much less than the number of possible combinations of pipe diameters. Then it is straightforward to calculate all parameters related to each network scenario and obtain the optimization results. This optimization method is probably not the most efficient one, but it is able to find out the overall optimal network design considering the two main factors of initial investment and pumping cost.

4. Conclusion

In this paper, we develop a multi-objective optimization method for district heating (DH) network design. We consider two main influencing factors, the initial investment of laying the pipes and the pumping cost. The initial investment will have a great impact on the construction of the network and the pumping cost represents the main operating cost after the construction. However, these two factors are conflicting objectives; if we use big pipes in the network, then the pumping cost is lower but the initial investment will increase dramatically and vice versa. In the optimization model, we generate all possible network scenarios consisting of different series of diameters for each pipe in the network. There are many possible diameters for one pipe disregarding the upstream and downstream pipes, but the series of diameters for one network scenario is strongly constrained by the two rules as follows: 1) the diameters of upstream pipes are larger or equal to the consecutive downstream pipes in the supply main line, and the order is reversed in the return main line; 2) the supply and return main lines are symmetrical. Then for each DH network scenario, we will calculate the annuity of initial investment, the pumping cost and the total annual cost using different combinations of steel prices and pump operating time levels to indicate the uncertainty and variation of them. Finally, we can obtain the optimization results and find out the overall optimal design.

The optimization method is demonstrated in an example DH network and the optimal scope of *R* values is found from 44.9 Pa/m to 79.3 Pa/m which justify the recommended *R* values. But we conclude that on one hand *R* should be less than 100 Pa/m but on the other hand it should not be too small, otherwise the total annual cost will increase due to the large initial investment. In addition, we also find that the pumping cost percentage is always increasing along with the increasing *R* values. This percentage increases from 41% to 82%. That is to say, if we use too thin pipes in the DH network (*R* values are large) the pumping cost will increase very fast, which means that the pumping cost is a more important factor when designing a DH network and it should be given more considerations compared to the initial investment.

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Nomenclature

Letter symbols

a annuity factor of initial investment

c the specific heat capacity of water, 4186.8 J/(kg °C)

d pipe diameter, m

g gravity constant, 9.81m/s^2

G mass flowrate, kg/s

H the lift height of the pump, m;

I initial investment, Yuan

K the inner surface roughness, m

L length of pipes, m

n_0 the full power operating time of the pump, h

P pressure or pressure loss, Pa

P_e the electric motor power of the pump, W

P_a pump axis power, W

Q volume flow, m^3/s

Q_{load} heat load, W

R specific pressure loss, Pa/m

r interest rate

Greek symbols

ε relative pipe roughness

η pump efficiency

η_m the transmission coefficient

λ fraction factor of the inner pipe surface

μ dynamic viscosity coefficient, kg/(m s)

ρ water density, kg/m^3

τ_1', τ_2' the design supply and return water temperatures, °C

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