

A fuzzy-grey multicriteria decision making model for district heating system

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Abstract:

District heating (DH) is playing an indispensable role in the energy supply all over the world. The high share of DH based on combined heat and power (CHP) indicates the energy efficiency of the local heating systems. In the future, the optimal planning and design of a DH system should consider not only the techno-economic feasibility but also the capability to improve energy efficiency and environment protection. This means that single objective optimization model for the planning of DH system is limited in this regard. Therefore, a multicriteria decision making (MCDM) model for decision support on the planning and designing of DH system is developed in this paper. This is a typical problem with uncertainty and imprecision both in the criteria measurements and the weights. In view of this, we adopted the fuzzy set theory and grey relational analysis to develop a fuzzy grey multicriteria decision making (FG-MCDM) model for DH systems. Sensitivity analysis was also conducted to study the influence of weight vectors on the evaluation results. The model can take into account energy, economy and environment concerns synthetically and thus facilitates more judicious decision making on DH systems.

Keywords:

District Heating; Combined Heat and Power; Multicriteria Decision Making; Fuzzy; Grey; Weight.

1. Introduction

International energy agency (IEA) reported that heating and cooling account for 46% of the total global energy use in 2012 [1], and district heating (DH) is becoming increasingly important for providing better comfortable conditions and services in buildings. More energy is to be consumed for heating, for example over 36% of the total building energy demand is consumed for residential heating in China in 2009 [2]; the situation is similar in Europe, e.g. in Finland the share of space heating in relation to the total end use of energy was 21% in 2005 and it gradually increased to 25% in 2012 [3,4], even though the specific energy consumption of DH is gradually reduced according to Helsinki Energy company. Nevertheless, under the circumstance of ongoing worldwide economy recession, decision makers (DMs), policy makers, stakeholders and end users have to face the problem of how to make the energy supply more cost-efficient, not only by human behaviours but also by system optimization. Meanwhile, environmental concerns, and the concept of eco-sustainability are widespread in the DH sector [5]. For these reasons, DH should be more energy, economic and environmental efficient in order to promote sustainable technologies and judicious decision making under different situations.

However, different heating have their own characteristics in relation to a host of criteria. Therefore, multicriteria decision making (MCDM) [6] methods should be used in the decision support for DH system. MCDM is a general term for methods that provides a systematic quantitative approach to support decision making in problems involving multiple criteria and alternatives [7]. The aim is to help the DM make more consistent decisions by taking important objective and subjective factors into account. The implementation of MCDM in DH can be helpful in the following aspects: 1) planning or retrofitting of DH system; 2) evaluation and selection of heat production technologies & DH scenarios and 3) optimization of the design, operation & regulation of DH system.

There are many MCDM methods for example summarized by Wang et al. [8,9], Pohekar and Ramachandran [10]. Some of the methods are suitable in energy planning, for example, outranking models ELECTRE [11] and PROMETHEE [12] have been applied in the evaluation and planning of DH systems in North America. In addition, other methods have also been widely used in the heating sector, such as multiple objective optimization (MOO) [13], linear programming (LP) [14], analytic hierarchy process (AHP) [15-16], multi-attribute utility theory (MAUT) [17] and so on. It can be found that MCDM methods are becoming more extensively used and the abovementioned models work well for specific problems, but not all of them are suitable for decision support of DH systems due to the coexistence of the following issues: 1) the selected evaluation criteria cannot reflect the key points of the problem; 2) misuse of data, e.g. qualitative indicators are used even when quantitative values can be originally obtained; 3) few of the above studies deal with uncertainties of weighting.

In fact, the MCDM of DH system is a typical problem with incomplete information, because some criteria are ordinal and cannot be measured precisely and others cannot be determined accurately or even missing. Therefore, this is a problem characterized by ‘fuzziness’ and ‘greyness (or incompleteness)’ that should be carefully addressed. Fuzzy mathematics [18] and grey system theory [19] provide powerful tools to deal with the two imprecise properties. For this reason, we integrate the fuzzy set theory and grey relational analysis (GRA) together and develop a MCDM model for decision support of DH system, named fuzzy-grey multicriteria decision making (FG-MCDM) model. The FG-MCDM model configuration is detailed in section 2. Section 3 shows an application of the model in planning of a combined district heating system. We also present the sensitivity analysis to show how the weight can affect the MCDM conclusions and provide a further discussion about the results. Finally we end the paper with overall conclusions in section 4.

2. Methods

2.1 Fuzzy set and triangular fuzzy number (TFN)

The multicriteria decision making in DH sector is affected by the uncertainty and imprecision information, which is suitable to be addressed by the fuzzy set theory introduced by Zadeh [20]. Specifically, a fuzzy subset A is defined by a membership function $f_A(x)$, which indicates the degree of x in A . The degree to which an element belongs to a set is defined by the value between 0 and 1. If x fully belongs to A , $f_A(x) = 1$, and fully not, $f_A(x) = 0$; the higher $f_A(x)$ is, the greater is the degree of membership for x in A . There are many kinds of membership functions with different shapes, in this paper we introduce the triangular fuzzy number (TFN) M [8] using a piecewise linear membership function defined in (1),

$$f_A(x) = \begin{cases} 1 & x = m \\ \frac{(x-l)}{(m-l)} & l \leq x < m \\ \frac{(r-x)}{(r-m)} & m < x \leq r \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

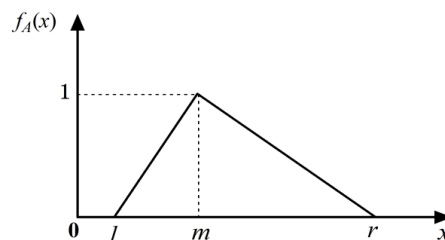


Fig. 1. The membership function of triangular fuzzy number M .

where $m \in [l, r]$, l and r are the upper and lower bounds of the triangular fuzzy number, and then TFN M can be expressed by (l, m, r) . $(r-l)$ implies the degree of fuzziness of a TFN, shown in Fig. 1. If $r=l$, then the TFN M degenerates to a real number m and the degree of fuzziness is thus zero. With TFN, we can describe the uncertainties in the MCDM as intervals instead of real numbers, which overcome the shortcomings of using deterministic values to represent uncertain criteria. Besides, basic manipulation laws should be mentioned to facilitate the use of TFN in the MCDM of DH system.

Let $M_1 = (l_1, m_1, r_1)$ and $M_2 = (l_2, m_2, r_2)$ are two TFNs, then [8],

(1) TFN summation: $M_1 \oplus M_2 = (l_1 + l_2, m_1 + m_2, r_1 + r_2)$;

(2) TFN multiplication: $M_1 \otimes M_2 \approx (l_1 l_2, m_1 m_2, r_1 r_2)$;

(3) TFN division: $\frac{M_1}{M_2} \approx \left(\frac{l_1}{r_2}, \frac{m_1}{m_2}, \frac{r_1}{l_2} \right)$;

(4) TFN multiplies real number: $\forall \lambda \in R, \lambda M_1 = (\lambda l_1, \lambda m_1, \lambda r_1)$.

(5) Ranking of TFN: $R(M) = (l+4m+r)/6$, $M_1 > M_2 \Leftrightarrow R(M_1) > R(M_2)$, $M_1 = M_2 \Leftrightarrow R(M_1) = R(M_2)$.

2.2 Grey relational analysis (GRA)

In 1980s, grey system theory was developed by Professor Julong Deng and has been widely used in many fields afterwards [19]. It is a suitable method to unascertained problems with ‘few data’ or ‘poor information’. Grey relational analysis (GRA) is a system analysis method for problems having multiple influencing factors. It can indicate the relevance of the system structure quantitatively and thus reveal the relevance degree of each alternative to the ideal scheme. The grey correlation coefficient (GCC) is used to measure the relevance degree of each alternative to the ideal scheme, the larger GCC is, the closer one alternative is to the ideal scheme.

We need to calculate the distance between a pair of sequences or called schemes, in order to compare and evaluate their performances. Normalization is required to facilitate this comparison process. Let $\Delta_{\max}(x)$ and $\Delta_{\min}(x)$ be the maximum and minimum absolute difference between two schemes or two sequences of criteria performances x_i and x_j for alternative i and j , then,

$$0 \leq \Delta_{\min}(x) \leq \Delta_{ij}(x) \leq \Delta_{\max}(x), \quad (2)$$

Namely,

$$0 \leq \frac{\Delta_{\min}(x)}{\Delta_{\max}(x)} \leq \frac{\Delta_{ij}(x)}{\Delta_{\max}(x)} \leq 1, \quad (3)$$

where $\Delta_{ij}(x)$ is the distance between two schemes on criteria x . It can be concluded from (3) that the larger $\Delta_{ij}(x)/\Delta_{\max}(x)$ is, the weaker similarity of x_i and x_j is observed, and vice versa. Then, we can reverse $\Delta_{ij}(x)/\Delta_{\max}(x)$ and normalize it to the interval $[0, 1]$ by,

$$\frac{\Delta_{\min}(x) / \Delta_{\max}(x)}{\Delta_{ij}(x) / \Delta_{\max}(x)}, \quad (4)$$

But $\Delta_{\min}(x)$ can be zero at times, therefore (4) is then defined as,

$$\frac{\Delta_{\min}(x) / \Delta_{\max}(x) + \rho}{\Delta_{ij}(x) / \Delta_{\max}(x) + \rho} \square \xi_{ij}(x), \quad (5)$$

where $\rho \in [0, 1]$, and (5) can take form,

$$\xi_{ij}(x) = \frac{\Delta_{\min}(x) + \rho \Delta_{\max}(x)}{\Delta_{ij}(x) + \rho \Delta_{\max}(x)}, \quad (6)$$

where $\zeta_{ij}(x)$ is the GCC of alternatives i and j on criteria x . According to (6), ρ is able to control the impact of $\Delta_{\max}(x)$ on $\zeta_{ij}(x)$ and it is called the coefficient of distinction which is usually set to be 0.5.

2.3 The fuzzy-grey multicriteria decision making (FG-MCDM) model

2.3.1 Criteria aggregation

In a complicated MCDM problem with many influencing factors, it is better to establish a hierarchical structure criteria aggregation system. The hierarchy consists of several different criteria levels: the first level should be the objective level and the rest of the levels should show the criteria meanings from general to specific, and then followed by the scheme level. In our study, economy, technology, environment and energy have been chosen as first-level criteria, while each of them can be divided into corresponding second-level criteria with different properties, shown in Fig. 2.

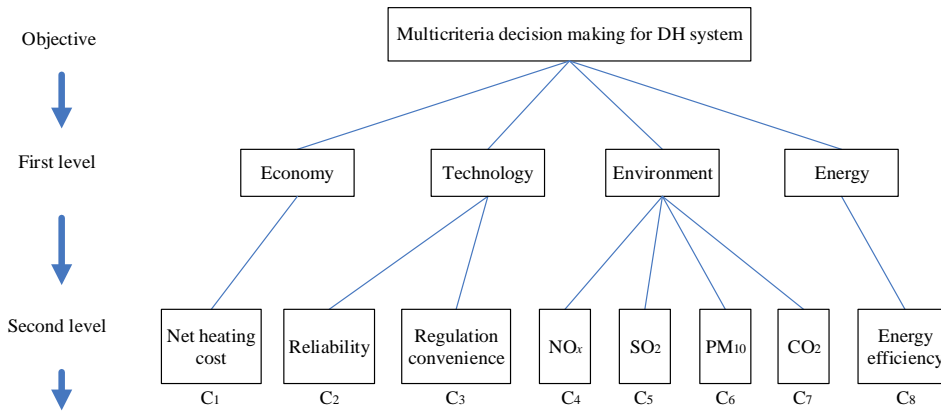


Fig. 2. The criteria aggregation hierarchy of multicriteria decision making for DH systems.

The current hierarchical structure is practical for the decision support of DH systems. Nevertheless, the criteria can be expanded or changed for some other heating technologies. The normalization of judgment matrices is conducted after obtaining criteria measurements.

2.3.2 Judgment matrix normalization

If a problem consists of m alternatives with respect to n criteria, the judgment matrix can take form,

$$G = [G_{ij}]_{m \times n} = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} G_{11} & G_{12} & \dots & G_{1n} \\ G_{21} & G_{22} & \dots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1} & G_{m2} & \dots & G_{mn} \end{bmatrix} \end{matrix}, \quad (7)$$

where A_1, A_2, \dots, A_m are alternatives; C_1, C_2, \dots, C_n are criteria and G_{ij} is the performance of alternative A_i on criterion C_j . Furthermore, if C_j is a cardinal criterion, then G_{ij} can be listed into the judgment matrix directly, otherwise we need to use TFNs, e.g. shown in Fig. 3 and Table 1 to describe the ordinal measurements. It is possible to use other fuzziness degrees for different problems.

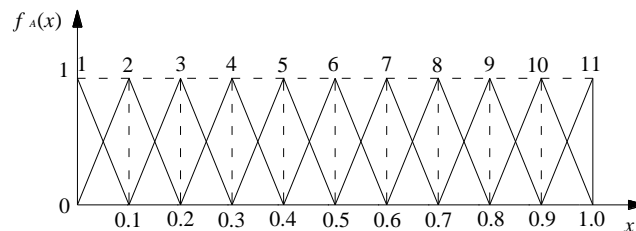


Fig. 3. The membership functions of TFNs denoting 11 different ordinal levels.

Table 1. TFNs for 11 different ordinal levels shown in Fig.3

Ordinal levels	1	2	3	4	5	6
TFNs	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.1, 0.2, 0.3)	(0.2, 0.3, 0.4)	(0.3, 0.4, 0.5)	(0.4, 0.5, 0.6)
Ordinal levels	7	8	9	10	11	
TFNs	(0.5, 0.6, 0.7)	(0.6, 0.7, 0.8)	(0.7, 0.8, 0.9)	(0.8, 0.9, 1.0)	(0.9, 1.0, 1.0)	

Before normalizing the judgment matrix, note that any real number can be interpreted as a TFN having a zero fuzziness degree. Therefore, it is convenient to denote the judgment matrix as an overall TFN matrix. On this basis, the normalization process can be uniformly carried out as follows.

If C_j is a positive (benefit) criterion, then,

$$R_{ij} = \frac{G_{ij}^+ - G_j^-}{G_j^+ - G_j^-} = \frac{[(g_{ij}^l - g_j^{l-}), (g_{ij}^m - g_j^{m-}), (g_{ij}^r - g_j^{r-})]}{[(g_j^{l+} - g_j^{l-}), (g_j^{m+} - g_j^{m-}), (g_j^{r+} - g_j^{r-})]} = \left(\frac{g_{ij}^l - g_j^{l-}}{g_j^{r+} - g_j^{r-}}, \frac{g_{ij}^m - g_j^{m-}}{g_j^{m+} - g_j^{m-}}, \frac{g_{ij}^r - g_j^{r-}}{g_j^{l+} - g_j^{l-}} \wedge 1 \right), \quad (8)$$

If C_j is a negative (cost) criterion, then,

$$R_{ij} = \frac{G_j^+ - G_{ij}^-}{G_j^+ - G_j^-} = \frac{[(g_j^{l+} - g_{ij}^l), (g_j^{m+} - g_{ij}^m), (g_j^{r+} - g_{ij}^r)]}{[(g_j^{l+} - g_j^{l-}), (g_j^{m+} - g_j^{m-}), (g_j^{r+} - g_j^{r-})]} = \left(\frac{g_j^{l+} - g_{ij}^l}{g_j^{r+} - g_j^{r-}}, \frac{g_j^{m+} - g_{ij}^m}{g_j^{m+} - g_j^{m-}}, \frac{g_j^{r+} - g_{ij}^r}{g_j^{l+} - g_j^{l-}} \wedge 1 \right), \quad (9)$$

$$G_j^+ = \max \{G_{ij}^+ | i = 1, 2, \dots, m\} = (g_j^{l+}, g_j^{m+}, g_j^{r+})$$

$$G_j^- = \min \{G_{ij}^- | i = 1, 2, \dots, m\} = (g_j^{l-}, g_j^{m-}, g_j^{r-}) \quad (10)$$

where, R_{ij} is the normalized performance of alternative A_i on criterion C_j ; $(g_{ij}^l, g_{ij}^m, g_{ij}^r)$ is the TFN for G_{ij} . It can be concluded that the R_{ij} is also a TFN, in which the r value is less or equal to 1.

2.3.3 Weight determination

AHP [21] is widely used to elicit the DMs' preferences and to compute the weight vectors. AHP has been updated constantly since it was first introduced. Currently, the "complementary judgment matrix (CJM)" has been introduced to AHP. In a CJM, two related pairwise comparison elements add up to a unit, that is, they add up to a complementary relationship rather than a reciprocal one. The main procedure for fuzzy AHP is similar to that for AHP. First, a complementary judgment matrix, A , should be constructed using the binary grading value shown in table 2. Then, a consistency check should be performed for all weighting judgment matrices, which can be found in Wang et al. [22]. Generally, only the judgment matrices that pass the consistency check can be used to calculate weight vectors. A complementary judgment matrix with n criteria can be written in (11).

Table 2. Binary grading value of complementary judgment matrix

Description	a_{ij}	a_{ji}
i th criterion is identical compared with j th	0.5	0.5
i th criterion is a little more important compared with j th	0.6	0.4
i th criterion is important compared with j th	0.7	0.3
i th criterion is very important compared with j th	0.8	0.2
i th criterion is extremely important compared with j th	0.9	0.1

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \quad (11)$$

where a_{ij} is the preference proportion of the i th criterion compared with the j th criterion. Assume that the weights of the i th and j th criteria are w_i and w_j , respectively. Then a_{ij} would take form,

$$a_{ij} = \frac{w_i}{w_i + w_j}, \quad (12)$$

Generally speaking, it is difficult to keep the complementary judgment matrices consistent. Therefore, a consistency check is necessary. However, if the inconsistency only varies slightly and can be deemed “satisfactorily consistent,” then the judgment matrix is still acceptable and can be used to calculate the weight vector by means of the weighted least square method (WLSM) [22].

2.3.4 FG-MCDM process

(1) Determine the criteria performance sequences for all alternatives and the reference sequences

The reference sequences are the artificial alternatives’ performances consisting of the best or worst criteria measurements among all alternatives in terms of each criterion and thus can be referred to as the best or worst reference sequence, which stand for the possible optimal alternative or the worst alternative in theory. They are denoted by Z_0 and Z_w ,

$$Z_0 = \{Z_{0j} | j = 1, 2, \dots, n\}, \quad (13)$$

$$Z_w = \{Z_{wj} | j = 1, 2, \dots, n\}, \quad (14)$$

Other sequences are the criteria performances for each alternative,

$$Z_i = \{Z_{ij} | i = 1, 2, \dots, m; j = 1, 2, \dots, n\}, \quad (15)$$

(2) Calculate the grey correlation coefficient (GCC) matrix

The GCC between a criteria performance sequence and the best sequences is,

$$\alpha(Z_{0j}, Z_{ij}) = \frac{\min_i \min_j |Z_{0j} - Z_{ij}| + \rho \max_i \max_j |Z_{0j} - Z_{ij}|}{|Z_{0j} - Z_{ij}| + \rho \max_i \max_j |Z_{0j} - Z_{ij}|}, \quad (16)$$

The GCC between a criteria performance and the worst sequences is,

$$\beta(Z_{wj}, Z_{ij}) = \frac{\min_i \min_j |Z_{wj} - Z_{ij}| + \rho \max_i \max_j |Z_{wj} - Z_{ij}|}{|Z_{wj} - Z_{ij}| + \rho \max_i \max_j |Z_{wj} - Z_{ij}|}, \quad (17)$$

where ρ is the coefficient of distinction, $\rho = 0.5$.

(3) Calculate the grey correlation degree (GCD) considering the weight vectors

The GCD between a criteria performance sequence and the best sequences is,

$$\bar{\alpha}(Z_0, Z_i) = \sum_{j=1}^n \alpha(Z_{0j}, Z_{ij}) w_j, \quad (18)$$

The GCD between a criteria performance sequence and the worst sequences is,

$$\bar{\beta}(Z_w, Z_i) = \sum_{j=1}^n \beta(Z_{wj}, Z_{ij}) w_j, \quad (19)$$

where w_j is the weight of criterion j . Therefore, the overall GCD of Z_i and Z_0 is defined as,

$$r(Z_0, Z_w, Z_i) = \frac{1}{1 + [\bar{\beta}(Z_w, Z_i) / \bar{\alpha}(Z_0, Z_i)]^2}. \quad (20)$$

It can be seen from (20) that the larger $r(Z_0, Z_w, Z_i)$ (overall GCD) is, the higher rank that alternative i receives. Therefore, we can use the overall GCD to rank the alternatives as it can indicate the distances from one alternative to the theoretical best and worst schemes comprehensively.

3. Results and discussion

3.1 The example combined district heating system

A combined district heating system is a kind of widely used DH system that has two or more heat sources, including base and peak heat sources in the same network. For a combined district heating system, an important parameter named base heat load ratio (β) [23] which indicates the ratio of base load and the design heat load of the whole system should be highlighted.

In this study, we demonstrate the FG-MCDM model for decision support by assessing combined district heating alternatives having different base heat load ratios in a DH system of a city in China. The base heat plants are CHPs and peak heat sources are gas-fired boilers. Some relevant parameters of this combined DH system are shown in Table 3 and more detailed data can be found in Wang et al. [5,23]. Eleven combined district heating alternatives were evaluated with different base heat load ratios ranging from 0.5 to 1, shown in Table 4. In fact, these alternatives are the schemes at the bottom of the criteria aggregation system in Fig. 2. Therefore, we have eleven combined district heating alternatives to be evaluated in terms of eight criteria.

Table 3. Design parameters of the combined district heating system

Item	Value	Unit
Heat load	616	MW
Heating area	8.6	million m ²
Heating substations	50	
Specific fractional resistance of main pipelines	30–70	Pa/m
Local resistance rate	30	%
Design supply and return water temperature	130/80	°C
Design outdoor temperature	−26	°C
Design indoor temperature	18	°C
Heating period	181	d

Table 4. Criteria measurements at different base heat load ratios

First level criteria	Economy	Technology		Environment				Energy
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
Second level criteria	▼ quant.	▲ quant.	▲ qual.	▼ quant.	▼ quant.	▼ quant.	▼ quant.	▼ quant.
	Net heating cost* (10 ⁸ Yuan)	Reliability ensuring coefficient (%)	Regulation convenience	C _{m_{sd}-NO_x} (μg/m ³)	C _{m_{sd}-SO₂} (μg/m ³)	C _{m_{sd}-PM₁₀} (μg/m ³)	CO ₂ emission (Mt)	Equivalent electricity (10 ⁸ kWh)
$\beta=0.50$	2.923	61.11	(0.0, 0.0, 0.1)	0.3601	0.1283	0.0371	2.025	25.215
$\beta=0.55$	2.657	55.00	(0.0, 0.1, 0.2)	0.4255	0.1553	0.0436	2.035	25.570
$\beta=0.60$	2.439	48.89	(0.1, 0.2, 0.3)	0.4801	0.1795	0.0495	2.044	25.888
$\beta=0.65$	2.269	42.78	(0.2, 0.3, 0.4)	0.5334	0.2008	0.0543	2.052	26.169
$\beta=0.70$	2.157	36.67	(0.3, 0.4, 0.5)	0.5349	0.2027	0.0543	2.059	26.413
$\beta=0.75$	2.085	30.56	(0.4, 0.5, 0.6)	0.5583	0.2137	0.0566	2.066	26.620
$\beta=0.80$	2.086	24.44	(0.5, 0.6, 0.7)	0.5760	0.2221	0.0583	2.071	26.792
$\beta=0.85$	2.115	18.33	(0.6, 0.7, 0.8)	0.5890	0.2282	0.0595	2.074	26.927
$\beta=0.90$	2.195	12.22	(0.7, 0.8, 0.9)	0.6008	0.2340	0.0607	2.077	27.026
$\beta=0.95$	2.333	6.11	(0.8, 0.9, 1.0)	0.5986	0.2377	0.0611	2.079	27.091
$\beta=1.00$	2.507	0	(0.9, 1.0, 1.0)	0.5986	0.2394	0.0601	2.080	27.122

*Chinese currency RMB, 1US dollar \approx 6.2 RMB Yuan at present.

In Table 4, there are seven quantitative criteria and one qualitative criterion described by TFNs. In order to determine the judgment matrix, sub-models should be employed to obtain measurements for quantitative criteria, for example, techno-economic appraisal model to minimize the net heating cost can be found in [23]; reliability ensuring coefficient that indicates the system reliability under the worst possible accident situation is introduced in [24]; environmental related issues are discussed in [5] by modelling the MSD (Mean Spatial Distribution) of each pollutant; the concept of equivalent electricity is based on calculating the total energy consumption (in electricity equivalent) considering the qualities of different energy forms. Consequently, the normalized criteria judgment matrix used in the FG-MCDM model can be calculated according to (8)-(10).

3.2 Weight analysis

The complementary judgment matrices for the example combined district heating system were obtained by questionnaire survey. It is difficult to make respondents understand the process of giving a judgment matrix, but valid feedbacks were still collected. Subsequently, these judgment matrices were used to compute the weight vectors using the CJM method. The weight vectors of first and second-level criteria with respect to the optimization objective is shown in Table 5.

Table 5. Average weights of the first and second-level criteria (%)

Criteria	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
First level criteria	42.55	11.43		18.18				27.84
Second level criteria	42.55	8.00	3.43	3.90	5.87	5.87	2.54	27.84

Table 5 shows that the net heating cost (C₁) is the dominant factor with a weight of 42.55% followed by the energy efficiency criterion (C₈), which has a weight percentage of 27.84%. The remaining criteria have weight percentages lower than 1/8. Criteria weights provide the interface with which DMs' preference can be reflected in the model, but introduce the subjectivity to the model at the same time. This problem can be handled with the sensitivity analyses in section 3.3.

3.3 Results and sensitivity analysis

It is widely acknowledged that the weight information can have great impact to the multicriteria evaluation results. Therefore, we propose a sensitivity analysis on the weight vectors, and check the influences of different weight vectors on the evaluation results. In the sensitivity analysis, we used five extreme weight combinations in addition to the initial weight vector. They are: 1) even weight distribution; 2) economy only; 3) technology only; 4) environment only; and 5) energy only. These weight combinations are shown in Table 6. The second-level criteria weights for all weight combination scenarios can be calculated based on Table 5 and shown in Fig. 4.

Table 6. Weight combination scenarios for sensitive analysis

Weight combinations	First-level criteria			
	Economy	Technology	Environment	Energy
Initial weight vector	42.55%	11.43%	18.18%	27.84%
Even weight	25%	25%	25%	25%
Economy only	100%	0	0	0
Technology only	0	100%	0	0
Environment only	0	0	100%	0
Energy only	0	0	0	100%

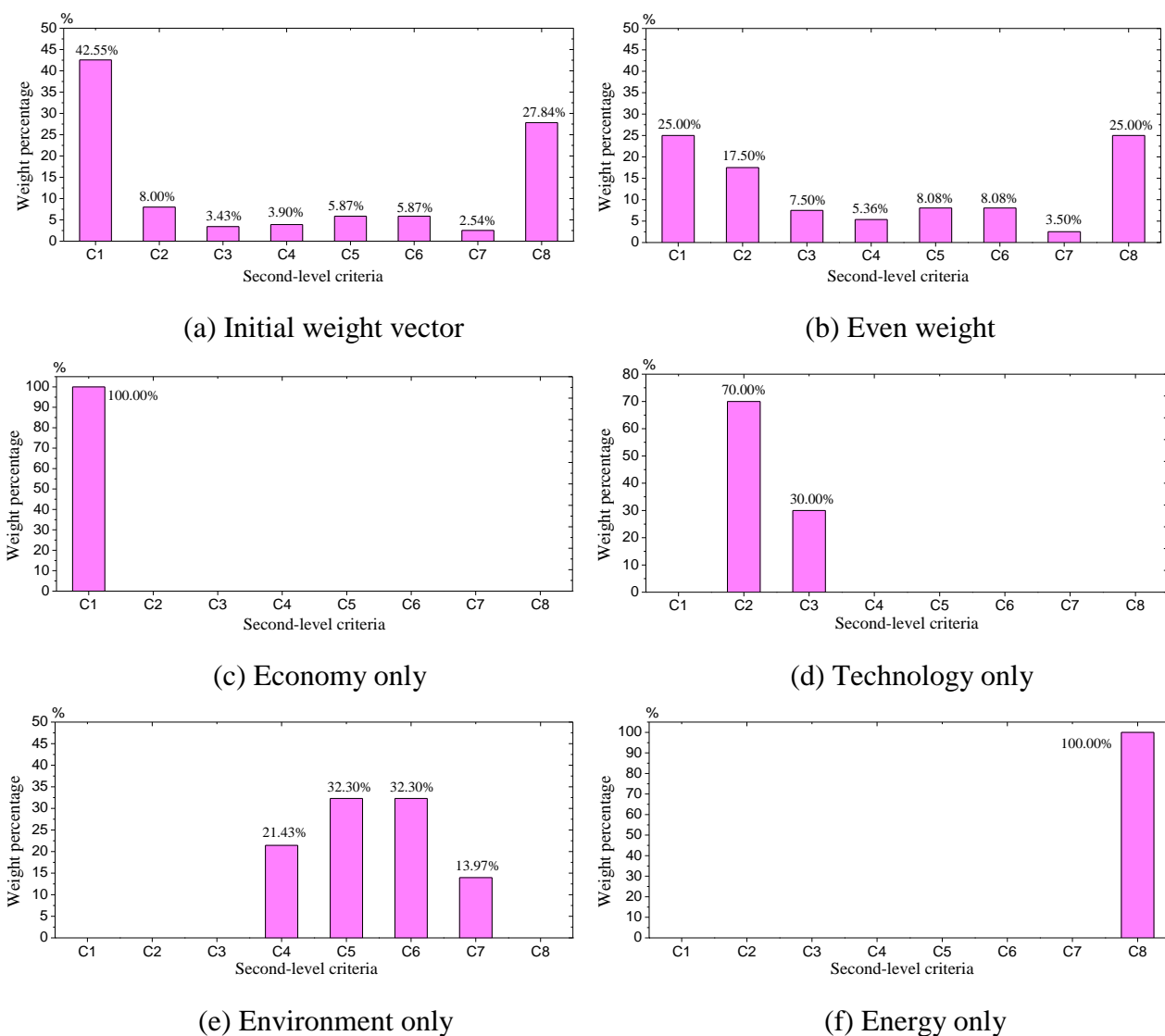
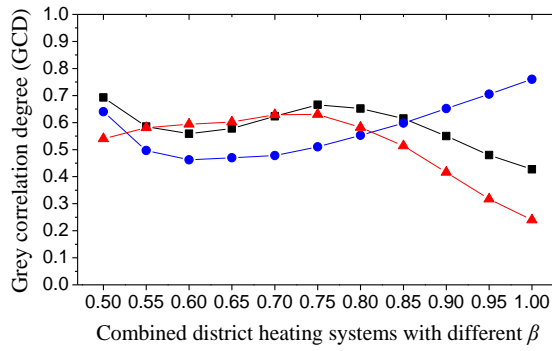


Fig. 4. Weight combination scenarios used in the sensitivity analysis.

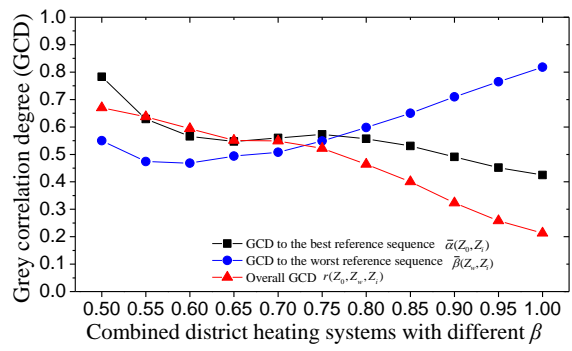
Figure 5(a) shows the FG-MCDM results corresponding to the initial weight vector. It was seen that GCDs of different combined district heating alternatives to the best and worst reference sequences have the same variation pattern before $\beta=0.75$; but they are developing towards different directions afterwards, i.e. getting away from the best sequence and approaching the worst sequence, leading to very low overall GCDs. Then we found that the overall GCD firstly increase slightly along with β and reach the maximum at $\beta=0.75$, after which it decreases dramatically. Namely the combined district heating alternative with $\beta=0.75$ is the most preferred scheme with initial weight vector.

It can be concluded from Fig. 5 that, the weight vectors have great influences on the multicriteria evaluation results. We found that unless the economy criterion are extremely favored, the combined district heating alternative with $\beta=1.00$ is the worst one. Even though the economy are the only criterion, $\beta=1.00$ is not the best alternative, instead $\beta=0.75$ is the optimal choice in Fig. 5(c). This means that the peak heat sources are required in the DH system and the economy performance can be better. For the weight combination scenario (b), (d), (e) and (f), the overall GCD is decreasing with the increasing value of β , which means that the combined district heating system is good in technology, environment and energy criteria, if the gas-fired boilers are used as the peak heat sources. It can also be found that the variation of GCDs to the best and worst references are not opposite all the time, on the contrary they have the same variation pattern sometimes, e.g. before $\beta=0.75$ in Fig. 5(a) and (b). Therefore, it is not reasonable to consider either only the GCDs to the best or to the worst references, instead we should consider the overall GCD to evaluate the alternatives. This is

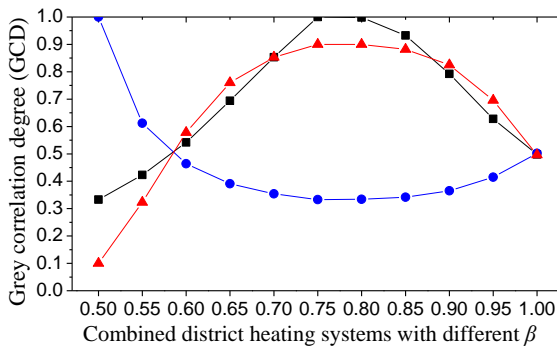
because we need to find the optimum which is most far from the worst reference and most close to the best reference at the same time.



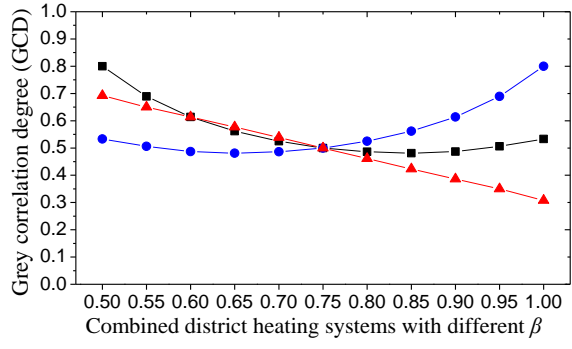
(a) Initial weight vector



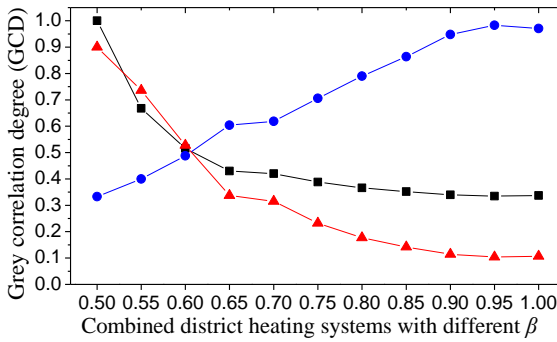
(b) Even weight



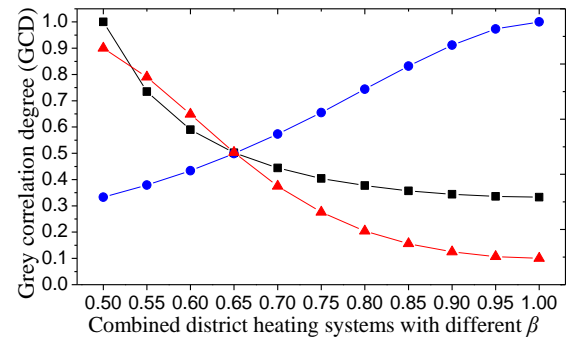
(c) Economy only



(d) Technology only



(e) Environment only



(f) Energy only

■ GCD to the best reference sequence $\bar{\alpha}(Z_0, Z_i)$ ● GCD to the worst reference sequence $\bar{\beta}(Z_w, Z_i)$ ▲ Overall GCD $r(Z_0, Z_w, Z_i)$

Fig. 5. Sensitivity analysis on weights of the combined heating system's FG-MCDM results.

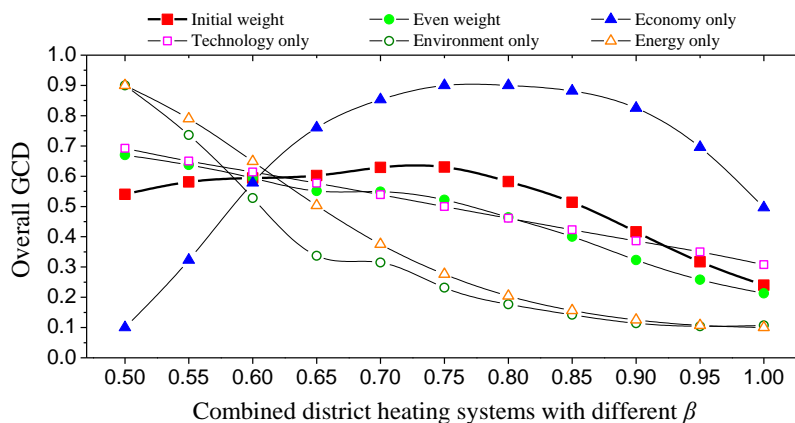


Fig. 6. Overall GCDs for combined DH scenarios with all weight combination scenarios.

Figure 6 shows the overall GCDs for all weight combination scenarios. It is interesting that the combined district heating alternative with $\beta=0.60$ is the most insensitive alternative whose overall GCD changes very little even with different extreme weight vectors. In addition, its overall GCD varies around a high value of 0.6, which means that the combined district heating alternative with $\beta=0.60$ is a quite good compromise alternative if DMs' preferences differs too much.

4. Conclusion

In this paper, we developed a fuzzy grey multicriteria decision making (FG-MCDM) model for the district heating (DH) system. In the model, we integrated the fuzzy set theory and grey relational analysis (GRA) method because the MCDM of DH system is a typical problem with uncertain or imprecise information. Therefore, we used triangular fuzzy numbers (TFNs) with some fuzzy degrees to describe criteria uncertainties and performed sensitivity analysis of extreme weight combinations to examine how the weight vectors can affect the multicriteria evaluation results. Then the FG-MCDM model was demonstrated in planning a combined district heating system in a city of China. In the case study, economy, technology, environment and energy criteria are considered. It is concluded that the developed FG-MCDM model works well for planning the district heating systems. In addition, it is not reasonable to consider either only the grey correlation degrees (GCDs) to the best or to the worst references, instead we should consider the overall GCD to evaluate the alternatives. This is because we need to find the optimal alternative which is most far from the worst reference and most close to the best reference at the same time. Further, according to the sensitivity analysis of weight vectors on overall GCDs of different combined district heating alternatives, the optimal alternative under initial weight vector is the combined district heating alternative with base heat load ratio equal 0.75, but the combined district heating alternative with base heat load ratio of 0.60 is a quite good compromise alternative if there is no weight information or DMs' preferences differs too much. Because it is the most insensitive alternative who has a relatively high overall GCD which changes very little even with different extreme weight vectors.

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