Finding Correlations Between the Crude Oil Price and Stock Market by Partial Wavelet Coherence

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Abstract:

Researches on relationships between the crude oil price and the stock market could help us to unspool the interaction between the international crude oil market and the economy of one country. Generally, time series analysis method is adopted to investigate the linkages between the crude oil price and the stock returns. However, the variation and diversity of the relationship among time series are still hidden in the time-frequency domain. Aiming to explore the linkages of the crude oil price, stock price and exchange rate from the joint time-frequency aspect, we examine the wavelet coherence between the Brent oil price and Chinese stock index, simultaneously exploring the partial wavelet coherence of them after removing the influence of the exchange rate. The results show that the coherence between the crude oil price and stock market are different for different time and frequency. Specifically, in the high and medium frequency band, the China stock index is leading the Brent during the period in which the Brent oil price changes dramatically and the exchange rate has marginal effect on their relationship. In the low frequency band, the Brent is leading the China stock index from 2006:10 to 2011 and the exchange rate enlighten the coherence between them since 2012.

Keywords:

Partial wavelet coherence, Crude oil price, Stock returns; Exchange rate.

1. Introduction

With the acceleration of global market integration and finalization of the commodity markets, the linkages between the commodity markets and the financial markets become increasing closer [1]. In particular, as strategy energy source and one of the most widely traded commodities, the fluctuation of the international crude oil markets has been closely related with the financial markets, namely the stock markets [2-5]. Moreover, during the international market, exchange rate is used to convert the currencies between the countries. Hence, proper understanding of the relationship among the intentional crude oil price has become the general concern of the researchers. There is a large body of the literature focus on this issue. For the oil-stock nexus there are mainly two kinds of results. The first one is that significant relationships exist between the crude oil price and domestic stock markets [6-8]. Second, Filis et al. and Park and Ratti [9, 10] proved that whether the crude oil price shocks impact the stock markets depends on various factors, namely the oil price increase driven by demand or supply side, the sampled country belongs to oil importer or exporter. In terms of the relationship between crude oil price and exchange rate, the results could be categorized into two kinds. One is that there is significant relationship between the crude oil price and exchange rate [11-14]. The other one is that the relationship between the exchange rate and crude oil price is uncertainty [15-17]. When it comes to the stock-exchange nexus, some researches prove that the stock markets and the exchange rate has close relationship and this relationship may change due to

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different time and countries[18, 19]. It is obvious that the international oil price, stock price and the exchange rate are linked in a complex way, which lead to consistent results.

In the above existing literature that focus on the relationships of the crude oil price, stock markets and exchange rate, the econometric model widely and generally adopted to carried out to analysis the time series in holistic aspect for the whole sample period. However, the holistic analysis in time domain hides the variation and diversity of the relationship among original time series. One hand, the relationship of the crude oil price, stock markets and exchange rate fluctuation and vary with time[20, 21]. In other words, for different sub-period their relationship may be various. On the other hand, the relationship of them also changes with different frequency components. The financial time series, namely, crude oil price, stock price and exchange rate, are consist of various agents with objectives from different time horizon, which lead the these financial time series are the combination of different frequency components that are corresponding to different time horizon. Hence, the analysis carried out in the time-frequency domain, which enables us examine the variation of the relationship different frequency components without losing the time information.

For the issue of examining the relationship in the joint time-frequency domain, the continuous wavelet transform offer us a novel and effective tool in the economic field. The main idea of the continuous wavelet transform is that the time series is represented by the wavelet function with two parameters of time and frequency, which enables us decompose the original time series into the joint time-frequency domain. Specifically, the wavelet coherence and partial wavelet coherence based on the continuous wavelet transform. The work principles of the wavelet coherence and partial wavelet are similar to the traditional correlation and partial correlation conception and extend these traditional conceptions into the time-frequency domain. Observing the variation of the coherence and partial coherence for different time and frequency could offer more insight to the research issue of the relationship of the crude oil price, stock market and exchange rate.

This paper is organized as following, in section 2 we demonstrate the sample and methodology of the wavelet coherence and partial wavelet coherence. Then, the empirical results and the discussion are presented in the section 4. Lastly, we will draw the conclusion in section 4.

2. Data and methodology

In this paper, we will explore the interaction between the Chinese stock market and the international crude oil price in the joint time-frequency domain. First we use the wavelet coherence to depict the relationship between the stock market and the price of crude oil in bivariate framework. Second, we will examine their relationship after removing the influence of the exchange rate with the partial wavelet coherence. Then, compare the results from the wavelet coherence and partial wavelet coherence.

3.1 Data

To explore the dynamic and interaction between the Chinese stock market and the international prices of the crude after controlling the influence of the exchange rate in the joint time-frequency domain. We use the Shanghai composite index (SCI), Brent spot price (Brent) and the U.S. dollar against RMB exchange rate (Exchange). All the prices were collected on a daily basis during the period from 2006:10 to 2014:12. The SCI data were obtained from Wind database and the data of Brent and Exchange were extracted from the website of U.S. Energy Information Administration (EIA) and Federal Reserve Bank of New York.

3.2 Continuous Wavelet Methodology

The wavelet coherence and partial wavelet coherence are both based on the continuous wavelet transform. The detail is described as followings.

3.2.1 Continuous wavelet transform

The main idea of the continuous wavelet transform is that utilizing the wavelet as the band pass filter to the original time series. A wavelet is a square integrable function with real-value and zero mean, in which there are two parameters namely, location (u) and scale (s). The location parameter s could determine the position of the wavelet in time by shift the wavelet, while the scale parameter u could stretch or dilate the wavelet to localize different frequency. And there is reverse relationship between the scale and the frequency, specifically the low scale correspond to the high frequency and vice versa.

$$\psi_{u,s} = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \tag{1}$$

According to the Heisenberg uncertainty principle, there is always a trade off between the localization of the time and scale. For the features extracting purpose, the Morlet wavelet with $\omega_0 = 6$ is a good choice because that it provides a good balance between time and frequency localization [22].

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2} \tag{2}$$

The continuous wavelet transform could be obtained by projecting the original time series onto the specific wavelet $\psi(\cdot)$ that is characterized by the location and scale parameters, which could be represented as the following equation.

$$W_X(u,s) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{s}}\overline{\psi\left(\frac{t-u}{s}\right)dt} \tag{3}$$

According to the continuous wavelet transform, we could attain further information about the time series, namely, amplitude and phase. First, the square of the amplitude $|W_X|^2$ is defined as the wavelet power spectrum, which indicates that the variance distribution of different frequency components of the original time series evolving in time, large variance corresponding to large power. Second, when given the wavelet function $\psi(\cdot)$ being complex, the wavelet transform W_X will also be complex, which consists of real part ($\Re\{W_X\}$) and imaginary part ($\Im\{W_X\}$). The phase could be defined as $\tan^{-1}\left(\frac{\Im\{W_X\}}{\Re\{W_X\}}\right)$ and considered as the position of each frequency components of the original time series at specific time, ranging from $-\pi$ to π, which could be used to relationship analysis with two time series in further analysis.

3.2.2 Wavelet coherence and phase difference

The main idea of the wavelet coherence is similar to the traditional linear correlation; however, it distinguish itself by demonstrate the correlation of two time series in the joint time-frequency domain. The calculation of the wavelet coherence is based on the cross wavelet transform and wavelet power spectrum of each time series. The cross wavelet transform of two time series $x(t)$ and $y(t)$ could be defined as $W_n^{xy} = W_n^x W_n^{y*}$, W_n^{y*} is the complex conjugation of the wavelet transform of time series $y(t)$. Then the cross wavelet power could be given by $|W_n^{xy}|$, which could be used to depict the covariance of two time series in joint-frequency domain. Hence, the calculation equation of the wavelet coherence could be given as following:

$$R(x,y) = \frac{\left|S(s^{-1}W_n^{xy})\right|}{S(s^{-1}|W_n^x|)^{\frac{1}{2}} S(s^{-1}|W_n^y|)^{\frac{1}{2}}} \tag{4}$$

Where S means a smoothing process in the time and frequency simultaneously (more details could see.

While the phase of one time series could be used to demonstrate the position in frequency cycle of the time series, the phase difference of two time series depicts their relevant lead and lag relationship at specific time and frequency.

$$f_{x,y} = \tan^{-1}\left(\frac{\Im\{W_n^{xy}\}}{\Im\{W_n^{xy}\}}\right), \quad with f_{x,y} \in [-p,p] \tag{5}$$

The phase-difference ranges from $[-p,p]$. More specifically, the phase difference of zero indicates that two time series move together. $f_{x,y}$ Î $(0,\frac{p}{2})$ means the examined time series moving in phase and x leading y. Then $f_{x,y}$ Î $(-\frac{p}{2},0)$ indicates *y* being lead. A phase-difference of p (or $-p$) indicates an anti-phase relation. If $f_{x,y}$ Î $(\frac{p}{2},p)$ then *y* is leading. Time-series *x* is leading if $f_{x,y}$ Î $(-p,-\frac{p}{2})$ [23].

3.2.3 Partial wavelet coherence

Partial wavelet coherence works like traditional partial correlation that enables us to detect the coherence between two given time series controlling the influence of other series. The calculation of the partial wavelet coherence is based on the wavelet coherence. For example, multiple wavelet coherence of $y(t)$, $x_1(t)$and $x_2(t)$, among them $y(t)$ is considered as the independent one. The wavelet coherence between $y(t)$ and $x_1(t)$, $y(t)$ and $x_2(t)$ and $x_1(t)$ and $x_2(t)$ is given as:

$$R(y,x_1) = \frac{\left|S(s^{-1}W_n^{yx_1})\right|}{S(s^{-1}\left|W_n^{y}\right|)^{\frac{1}{2}}S(s^{-1}\left|W_n^{x_1}\right|)^{\frac{1}{2}}} \tag{6}$$

$$R^2(y,x_1) = R(y,x_1)\times R(y,x_1)^* \tag{7}$$

$$R(y,x_2) = \frac{\left|S(s^{-1}W_n^{yx_2})\right|}{S(s^{-1}\left|W_n^{y}\right|)^{\frac{1}{2}}S(s^{-1}\left|W_n^{x_2}\right|)^{\frac{1}{2}}} \tag{8}$$

$$R^2(y,x_2) = R(y,x_2)\times R(y,x_2)^* \tag{9}$$

$$R(x_2,x_1) = \frac{\left|S(s^{-1}W_n^{x_2x_1})\right|}{S(s^{-1}\left|W_n^{x_2}\right|)^{\frac{1}{2}}S(s^{-1}\left|W_n^{x_1}\right|)^{\frac{1}{2}}} \tag{10}$$

$$R^2(x_2,x_1) = R(x_2,x_1)\times R(x_2,x_1)^* \tag{11}$$

The partial wavelet coherence of $y(t)$ and $x_1(t)$ with controlling the effect of $x_2(t)$ can be written as,

$$RP^2(y,x_1,x_2)=\frac{\left|R(y,x_1)-R(y,x_2)\times R(y,x_1)^*\right|}{[1-R(y,x_2)]^2[1-R(x_2,x_1)]^2} \qquad (12)$$

The above equation is the squared results of the partial wavelet coherence of three time series, which explains that the proportion of wavelet power of the dependent time series $y(t)$ that contribute to the independent ones $x_1(t)$ with controlling the influence of $x_2(t)$ at specific time and frequency.

3. Empirical results and discussion

3.1 Comparison between pairs coherence and partial coherence

Wavelet coherence and partial wavelet coherence are used to detect the interaction between given time series in joint time-frequency domain. The main idea of the wavelet coherence is similar to the traditional correlation. The partial wavelet coherence extends the conception of the wavelet coherence to multiple variables frameworks, which reveal the coherence between two series after controlling the effect of the third series for different time and frequency. The results that come from the wavelet coherence and multiple wavelet coherence are visible and easy to interpret, which can offer more information to detect co-movement of time series from time-frequency aspect. In addition, we use the Monte Carlo simulation to assess the statistical significance of the correlation at specific time and frequency [24, 25]. Another point worth to be noted is that due to the inherent features, the continuous wavelet transform and relative transform has the edge effects. Therefore, cone of influence (COI) is introduced to identify the area influenced by the edge effects.

The results of the wavelet coherence and partial wavelet coherence are shown in Fig. 1 (b) and Fig. 1 (c). The horizontal axis depicts the time period from 2006:10 to 2013:12, while the vertical axis demonstrates the frequency ranging from scale 1 (1 day) to scale 512 (roughly 2 market years). The scale and frequency has reverse relationship, more specifically, the lower the scale, the higher the frequency. The results of the wavelet coherence and multiple wavelet coherence can detect the time and frequency interval in which the time series has high correlation, which circled by the black line with warmer colour. In contrast, the regions outside significant interval with colder colour means less dependence of the time series.

The results of wavelet coherence are shown in Fig. 1 (b), which enables us to detect the interaction between Brent and Shanghai composite index for different frequency components with time information. In addition, we also could interpret the phase relationship between given series through the phase arrows. In the 4-128 days frequency band, there are a few high coherence regions; two of them are worth to be noted. One localizes in 8-60 days frequency band through 2008 and another one is in 60-100 days frequency band from 2010 to 2011:6 are corresponding the periods in which the Brent oil price changed dramatically. The phase arrows indicate that the SCI leads the Brent in the two above mentioned high coherence regions. The decrease and increase of the Brent during period of 2008 and from 2010 to 2011:6 were both driven by the reason from the demand side. Hence, we can infer that the dramatic Chinese stock market perform could used to predict the international crude oil price changes, which could contribute to increasingly grown dependence on the foreign crude oil. Come to the frequency band of 300-512 days, the Brent and SCI are highly coherence through the whole observation period and Brent lead SCI. In the sampled time period, China as a net oil importer, its import volume of the crude oil has ascended to 310 million tons in 2014 and the dependence of the foreign crude oil is up to 59.6% simultaneously. This dependence of the foreign crude oil directly lead the increasing influence of the international crude oil price to the Chinese stock market.

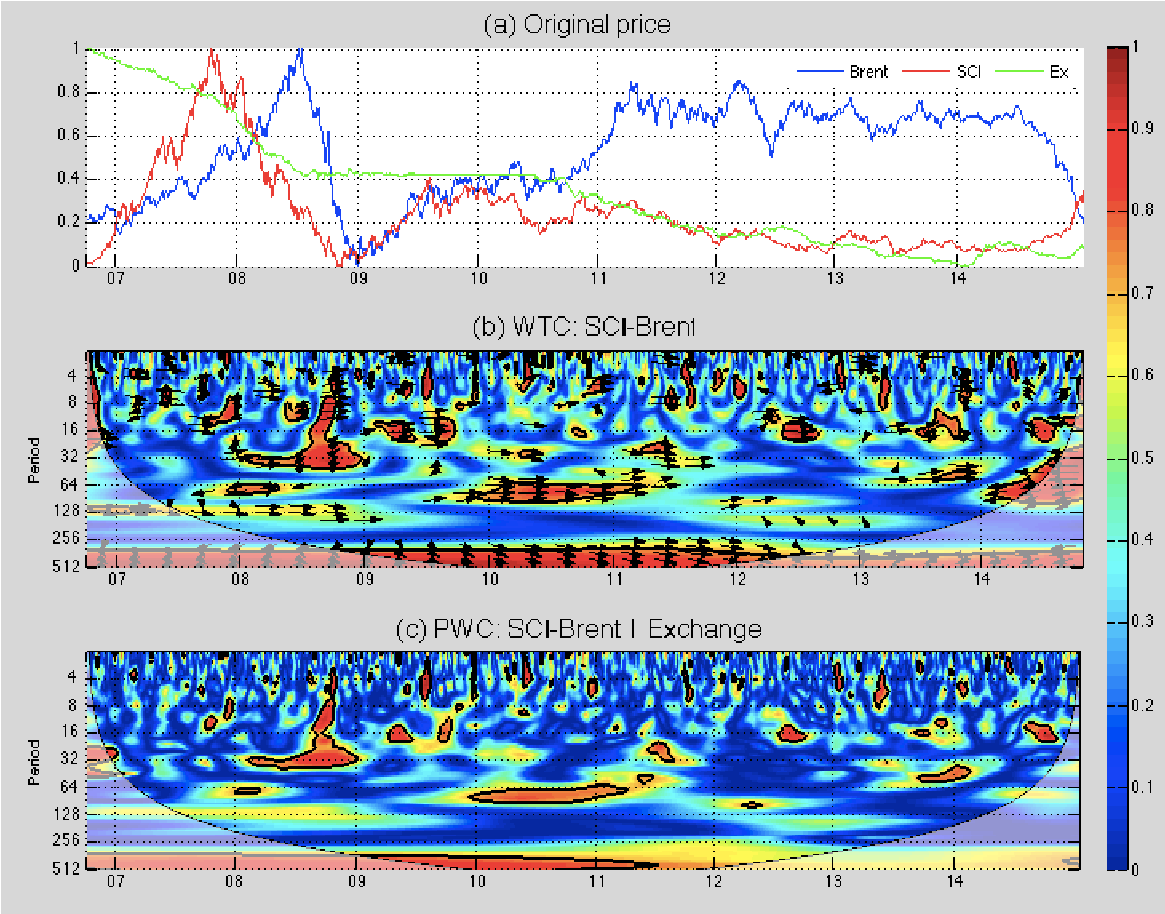

Fig. 1. The original prices, wavelet coherence results of the Brent and Shanghai security index and partial wavelet coherence results of the Brent and Shanghai security index removing the influence of exchange rate

The result of the partial wavelet coherence of SCI and Brent after controlling the effect of exchange rate is shown in Fig. 1(c). Compared with the results of the wavelet coherence in Fig. 1(b), in the frequency band of 1-128 days, the number and size of the high coherence regions decrease and is lightly smaller in partial wavelet coherence results, respectively. The frequency band of 1-128 days is corresponding to short and medium term. It is obvious that the interaction between the Brent and the Chinese stock market is affected by the exchange rate marginally in the short and medium term. In contrast, in the frequency band of 256-512 days, after controlling the effect of the exchange rate, the high coherence region of SCI and Brent shrinks dramatically and disappears since 2012. The U.S. dollar against RMB exchange rate has been last decreasing during the sampled period and since 2011, which strengthen the purchasing power of RMB. During the crude oil international trade, the U.S. dollar is generally used as the invoicing currency, with the decreasing of the U.S. dollar against RMB exchange rate; the forex burden of China is enlightened, which makes the exchanges rate could exact more effect on the relationship between the Brent and SCI is enlightened.

3.2 Heterogeneity test

Through the comparison between the wavelet coherence of Brent-SCI and the partial wavelet coherence of Brent-SCI exclude the influence of the Exchange rate, we only can depicts the difference in general way and cannot make inference precisely. Hence, we calculate the correlation coefficients and difference between the wavelet coherence and the partial wavelet coherence. The wavelet coherence and the partial wavelet coherence are actually $n*m$ matrix, where n means time information and m means frequency information. The correlation between two matrixes can be calculated by the equation as follows,

$$r = \frac{\sum_m \sum_n (A_{mn} - \bar{A})(B_{mn} - \bar{B})}{\sqrt{\left(\sum_m \sum_n (A_{mn} - \bar{A})^2\right)\left(\sum_m \sum_n (B_{mn} - \bar{B})^2\right)}}, \tag{13}$$

where A_{mn} means the factor of matrix A in $m$row and n line, B_{mn} means the factor of matrix B in m row and n line. $\bar{A}$ and $\bar{B}$ mean the average value of matrix A and B, respectively. And the correlation between the wavelet coherence of Brent-SCI and the partial wavelet coherence of Brent-SCI exclude the influence of the Exchange rate is 0.8418, which supports our former results.
Then we discretize the partial wavelet coherence by choosing the frequency bands of 2 days, 4 days, 8 days, 16 days, 32 days, 64 days, 128 days, 256 days and 512 days. Summarizing the variance for each frequency band from 2006 to 2014 (Fig. 2). Generally, whit the increasing of the frequency, the variances for each frequency band also go down. But expect in the year 2006 and 2013, there are fluctuations in specific frequency band in other years, which indicates that the coherence under corresponding frequency band happen structure changes.

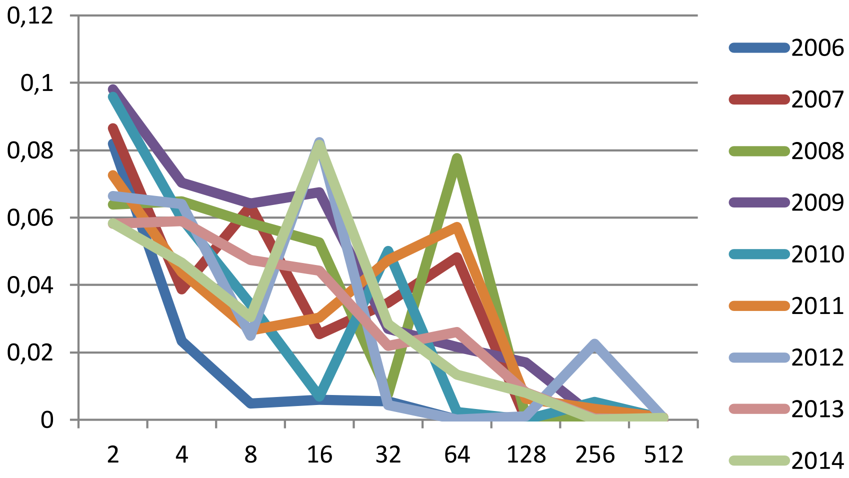

Fig. 2. The evolution of the variance for each frequency from 2006 to 2014

4. Conclusion

In this research, we aim to explore the coherence between the China stock market and the Brent oil price in the joint time-frequency domain. In this case, we could detect their interaction for different frequency components without losing the time information. Specifically, we use the wavelet coherence in bivariate framework and partial wavelet coherence in multiple variables framework. We found that in the period during which the Brent changes dramatically, the Brent and Chinese Stock market are highly correlated in the medium frequency band that is corresponding to medium term. Furthermore, in the period of Brent changes drove by demand reason, the Chinese stock market lead the Brent oil price. Based on these, we could use the information of the stock market to infer the following changes of the crude oil markets in the short and medium time horizon. When it comes to the 256-512 days frequency band, the exchange rate dropping below 6.4 strengthen the purchasing power of RMB, which enhanced its effect on the coherence between the Brent and Chinese stock market. Hence, diversity of the foreign oil support countries in which exchange rate with RMB is keeping decreasing is an effective way to lower the risk of the energy security in China. Furthermore, the precise correlation coefficient between the wavelet coherence of Brent-SCI and the partial wavelet coherence of Brent-SCI excluding the influence of exchange rate is 0.8418.

Except the exchange rate, there are more variables, etc., interest rate, industry production, and inflation rate could exert the influence on the relationship between the domestic stock market and the international crude oil price. Hence, in the further study, it is necessary to involve more factors to consider the oil-stock nexus, which will offer more insight to understand the economy system consist of a variety of agents.

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