Application of Genetic Algorithms in the Optimization of a Thermal System based on Micro-CHP Gas Turbines

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Abstract:

The combined heat and power production allows for the optimal use of primary energy sources and significant reductions in carbon emissions. Its use has a great potential for applications in the residential sector. Micro-gas turbines reveal several potential advantages for small-scale combined power generation, e.g., their compact size and low-weight per unit power, lower noise, multi-fuel capabilities, high-grade waste heat as well as high overall efficiency (in the CHP context). This study presents an application of Genetic Algorithms for the thermal-economic optimization of a small scale micro-gas turbine for cogeneration purposes. The system comprises an internal pre-heater, a combustion chamber, a turbine and an external heat exchanger able to fulfil hot domestic energy needs with a thermal power out of 125 kW. The thermal-economic model was developed considering the energy balance equations that describe the thermodynamic relationships between the components and five purchase cost equations were developed. A constrained non-linear optimization model was built up. The objective function was defined as the maximization of the annual worth from the CHP operation. This objective function represents the balance between the annual incomes and the expenditures subject to physical and economic constraints that gives significance to the problem. Regarding the decision variables, in the gas turbine, the inlet temperature and its isentropic efficiency are the most relevant decision variables, whereas, in the compressor, it is the compression ratio. A genetic algorithm coded in the java programming language was developed to solve the problem. The development of the genetic algorithm comprises the design of the fitness function, the selection of the population size and the definition of the limit-generation and crossover operators. A number of settings were tested and evaluated, e.g. encoding of the design, number of individuals in the population, mutation rate, elitist strategy and convergence criteria. The algorithm differs from the standard implementations in that it performs continuous population updates regardless of generation cycle completion as well as single gene cross-over operations to reduce the number of unfit offspring. Results have shown that optimal combination of decision variables disclose a system with higher performance output where the electrical efficiency is higher than the commercial models within the same energy output range, allowing for a positive primary energy saving. It has been shown that the optimal sizing of micro-gas turbines plays a key role in the overall economic viability.

Keywords:
Genetic Algorithms, Micro-gas turbine, Thermal-economic optimization, Micro-cogeneration.

1. Introduction

Recently, the decentralized power generation systems have resulted in remarkable results concerning the decrease in energy losses and the increase in reliability of energy supply. This way, local small power plants will further contribute in power generation, and in the not too distant future, they will replace large and centralized power plants [1,2]. By definition, cogeneration is the simultaneously generation of energy in different forms at optimum efficiency in a cost-effective and environmentally accountable way, allowing a reduction on primary energy consumption. In the last few years, this kind of technology has gained wide acceptance due to the high fuel prices, climate change awareness, compulsory guidelines for energy efficiency and favourable economic and political conditions. In Europe, the

Energy Performance of Buildings Directive (EPBD) and its recent recast [3] opened new opportunities for small-scale systems applied to the buildings sector. The directive obliges that, at the building design stage, the economic feasibility of high-efficiency alternative systems such as cogeneration is taken into account.

Furthermore, the Cogeneration Directive 2004/8/EC [4], which came into force in 2006, largely promoted the energy efficiency and the improvement of energy supply security. Its purpose is, basically, the creation of a framework for the development of high efficiency cogeneration, based on the useful heat demand of the customer and the Primary Energy Savings (PES).

The applications for cogeneration in the building sector include, for instance, hospitals, office buildings and single- and multi-family residential buildings. Specifically in the CHP applications for single-family, the design of the systems is associated with technical challenges due to the non-coincidence of thermal and electrical loads, requiring electrical/thermal storage or connection to the electrical grid [5]. CHP power plants are usually connected to the lower voltage distribution grids. Besides reducing losses in transmission and distribution, they can bring improvements to grid power quality, supplying energy when required. Moreover, applications in the residential sector offer opportunities in terms of improving energy efficiency and reduction of gas emissions.

It is well believed that technologies like Stirling engines and fuel cells are promising for small-scale cogeneration in the near future [6], because of their potential to achieve high efficiency and low emissions level. Nevertheless, those technologies are still in development and they are not available at reasonable cost [7,8]. So far, more mature technologies such as micro-gas turbines are already established in the market and have a great potential in combined heat and power appliances [9]. This technology is characterized by their higher efficiency in the cogeneration mode, lower pollution, lower commissioning time, compact size, higher runtime, and lower maintenance (due to lower movable parts). Micro gas-turbines are addressed as one of the most thoughtful options for CHP generation allowing overall efficiencies above 75% (based on Low Heating Value, LHV) with a total investment costs estimated to vary from 1000 to 1700 EUR/kWel [10].

Still, there is a space to optimize these thermal systems. Optimization is an important tool in the process of designing, implementing and testing algorithms for solving a large variety of real problems. The selection of the best algorithm to implement is deeply related to the objective function, the number of variables and the constraints that give significance to the physical problem and the smoothness of the functions (differentiable or not differentiable functions).

Several authors dedicate their investigation in the optimization and thermal-economic assessment of gas turbines for cogeneration. Datta et al. [11] presented exergy analysis of a coal-based thermal power plant by splitting up the entire plant cycle into three zones for the analysis. Ertesvag et al. [12] presented a concept for natural-gas (NG) fired power plants with CO_2 capture was investigated based on the exergy analysis. Natural gas was reformed in an auto-thermal reformer, and the CO_2 was separated before the hydrogen-rich fuel was used in a conventional combined-cycle process. The main purpose of the study was to investigate the integration of the reforming process and the combined cycle. An increase of the turbine in let temperature (TIT) from 1250 to 1350◦C and1450◦C increased the net electric-power production to 50.6% and 52.2%. One of the most interesting works regarding the optimization of gas turbines for CHP applications was the CGAM problem, which proposed a standard mathematical formulation for all thermo-economic models [13]. The CGAM problem was constructed to assess the optimum design parameters of a simple CHP apparatus that produces 30 MW of electricity and 14 kg/s of saturated steam. The purpose of this physical model was the standardization of thermo-economic methodologies through the common definitions of physical, thermodynamic and cost models plus the objective function [13]. Lazzareto and Toffolo also optimized the design parameters of the CGAM problem plant. Firstly the authors considered the energetic and economic objectives [14], and lately, a third objective function, which introduced environmental impact through the pollution related costs in the thermo-economic analysis [15]. They concluded that, if the design choice is guided by only economic considerations, it may not be a good choice in the limits set by restrictions imposed by environmental regulation. This model was the basis of the Silva et al. [16] work, who studied the optimization of CHP applications by adapting the CGAM problem to a small scale application. In their study it was performed a thermodynamic study of a cogeneration system based on a gas turbine applied to a local textile factory at the northwest region of Portugal. The model was developed considering a compressor/turbine rated at 5 MW_{el}, a boiler (10 bar) and a heat recovery unit for the thermal fluid. The problem was formulated as a non-linear optimization problem with constraints and it was solved with different numerical methods. More recently, Kong *et al.* [17] studied the optimal energy management of cogeneration system for combined cooling, heating and power production. In this study it was developed linear programming model to determine the optimal strategies that minimize the overall cost of energy for the CHP system (also with cooling). McDonald [18] and Vissi *et al.*[19] concluded in their studies that the major technical factors that challenge the development of micro turbines are related to the small-scale effects (e.g. large fluid dynamics, heat and mechanical losses) and costs. Other researchers dedicated to optimize a single component of the systems. Several studies can be found for the optimization of the internal pre-heater, a key component in the technological development of these systems [20,21].

Recently, several authors have proposed approximate methods, including heuristic to solve these optimization problems instead of using optimization methods, such as linear-programming or quadratic programming. Meta-heuristics are generalizations of heuristics in the sense that they can be applied to a wide set of problems, needing few modifications to be adapted to a specific case. In some cases, the complexity of the problems to solve is so high that even heuristic and meta-heuristic methods requires large runtimes to obtain accurate solutions [22]. The most used way to classify meta-heuristic algorithms is based on trajectory methods versus population-based methods. Population-based methods use a population of solutions which evolve during a given number of iterations, also returning a population of solutions when the stop condition is achieved. The main population-based meta-heuristics include: genetic algorithms (GA) and evolutionary algorithms (EA), particle swarm optimization (PSO), between others [23]. GA methods do not need to calculate the gradient of the fitness function and the constraints may be introduced in an augmented fitness function as penalty function as it is done with deterministic methods. Also, if population diversity is guaranteed, GAs are better suited for highly nonlinear optimization problems since they are more likely to avoid being trapped in local minima. In general, numerical estimation methods, such as maximum likelihood estimation (MLE), have several limitations pertaining to the existence of certain constraints imposed to estimation parameters of the objective function most often requiring continuity and the existence of derivatives. Valdés et al. [24] presented a study concerning the thermo-economic optimization of combined cycle gas turbine power plants using a genetic algorithm. Multi-objective optimization has also been carried out in thermo-economic analysis [15]. Ahmadi & Dincer [25] report a comprehensive thermodynamic and exergo-economic modelling of a Gas Turbine (GT) power plant, aiming to validate the thermodynamic model through a multi-objective optimization.

In this paper, the application of genetic algorithms for the thermal-economic optimization of small scale micro-gas turbine for cogeneration purposes is presented. A genetic algorithm coded in java language was developed to solve the non-linear optimization problem, by considering the maximization of the annual profit from the CHP system operation as the objective function. Six decision variables were chosen and several non-linear constraints were defined to give physical significance to the problem. In the section two of this paper, it is presented the application of the CHP system as well as the physical and thermodynamic system description. In section three, the formulation of the optimization model is defined, identifying the objective function, decision variables and inequality constraints. The fourth section comprises the design of the genetic algorithm (i.e. fitness function definition, selection of population, *etc*.) applied in the study. The last two sections correspond to the results and main conclusions, respectively.

2. Physical System

2.1. Application and System description

The problem presented in this study aims to optimize a small-scale cogeneration system by producing electrical power, and simultaneously be able to fulfil both the heating and the domestic hot water needs, for a building of residential apartments. The building consists of a 52 individual dwellings with an individual floor area of 150 m^2 (or 7800 m^2 in total). The annual thermal power duration curve of the building was calculated according to the Portuguese regulation for the thermal behaviour of buildings (RCCTE, Decree Law 78/2006) [26], by summing the hourly heating load and the hourly hot water needs. The domestic hot water needs calculations were performed considering an occupation of 4 people per dwelling with a daily domestic hot water consumption of 40 L per person, at a temperature of 333

K. The building hourly heating loads were calculated, considering a class B minus building and a local climate (i.e. north of Portugal). For this scale of application, the system must operate, approximately, 4000 h which corresponds to a heat output of 125 kW_{th}. Figure1 illustrates the layout of a micro turbine based CHP system. The turbo machinery and the electric generator are connected to a common shaft rotating at high speed. An inverter decouples the high frequency of the produced current from the grid, thus enabling variable speed operation. For so small applications with a low-pressure compressor, gas turbines require an Internal air Pre-Heater (IPH) or regenerator, to provide a satisfactory electrical efficiency. The thermal energy of the Exhaust Gases (EG) is recovered as useful heat. Atmospheric air is compressed (C) and fed to the IPH before entering the Combustion Chamber (CC) where it is mixed with Natural Gas (NG). The high temperature combustion gases expand in the Turbine (T). The EG, leaving the turbine, are firstly used in the IPH to pre-heat the incoming compressed air and subsequently for the production of hot water in the external heat recovery system, before exiting to the atmosphere. The latter is a Water Heat Exchanger (WHE), where a fixed flow rate of water is heated from 313 K to 353 K.

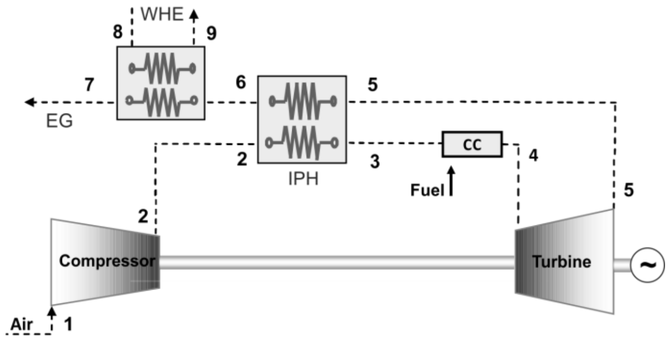


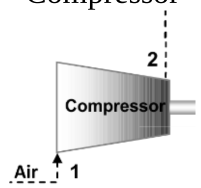
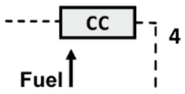
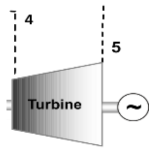
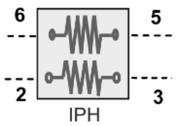
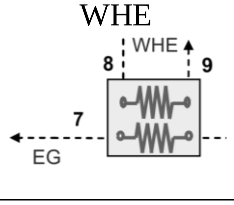
Fig. 1. Schematic layout of the micro-gas turbine CHP system.

For the physical model equations, the operating fluid properties were calculated through standard thermodynamic relationships. All the components were considered adiabatic and a degree of irreversibility was assumed for both the compressor and the turbine. The air and gases were treated as perfect gases with constant specific heats and the compressor inlet conditions were assumed as T_1=293 K and the atmospheric pressure (1.013 bar). A reasonable pressure drop is assumed for the flows through the IPH, CC and WHE.

2.2. Thermodynamic Modelling

Micro-turbines operate under the Joule-Brayton cycle. The power produced by the turbine and consumed by the compressor is proportional to the absolute temperature of the gas passing through them. Thus, it is advantageous to operate the expansion turbine at the highest practical temperature consistent with economic materials. For the thermodynamic modelling, the operating fluid properties were calculated through standard thermodynamic relationships and were fully explained elsewhere [27]. Nevertheless, the main relationships associated to the five plant components are listed below in Table 1. The compressor outlet temperature, T_2 , as in (1), is defined as a function of the compressor isentropic efficiency, η_C , the polytropic adiabatic coefficient, γ, and the compressor pressure ratio r_C. The mechanical power absorbed by the compressor is given by, $\dot{W}_C = \dot{m}_a \, c_{pa} \, (T_2 - T_1)$ and the air mass flow rate, $\dot{m}_a$ (as in (2)) which is calculated as a function of the net mechanical power, $\dot{W}$, the fuel to air mass ratio f and the C_{pa} and C_{pg} are the specific heats at constant pressure of air and flue gases, respectively. The fuel properties are those of the standard natural gas and a combustion efficiency of 98% is considered. The fuel mass flow rate, $\dot{m}_{fuel}$, can be estimated by the relationship, $\dot{m}_{fuel} = \dot{m}_a \, f$ with the fuel to air mass ratio calculated as in [27]. The gas mass flow rate, $\dot{m}_g$, resultant from the combustion of air and fuel is simply the sum of the air and fuel mass flow rates, $\dot{m}_g = \dot{m}_a + \dot{m}_{fuel} = \dot{m}_a \cdot (1+f)$. The gas mass flow rate, $\dot{m}_g$, resultant from the combustion of air and fuel is simply the sum of the air and fuel mass flow rates, $\dot{m}_g = \dot{m}_a + \dot{m}_{fuel} = \dot{m}_a \cdot (1+f)$. The exhaust gases, leaving the combustion chamber at the controlled maximum temperature T_4, are then expanded in the turbine and its exit temperature, T_5 is estimated by the relationship in (4) where the turbine outlet temperature varies with the turbine isentropic efficiency, η_T. The mechanical power delivered by the turbine is given by, $\dot{W}_T = \dot{m}_g\, c_{pg}\,(T_4 - T_5)$. As previous mentioned, the system includes an internal pre-heater to increase the micro-turbine electrical efficiency. This plant component is basically an air-gas heat-exchanger used to pre-heat the air before entering the combustion chamber (T_3). For a given heat-exchanger effectiveness ε_{IPH}, the T_3 can be calculated by $T_3 = T_2 + \varepsilon_{IPH}\,(T_5 - T_2)$. In the heat exchange between the air, T_2, and the expanded gases, T_5, it is assumed that the thermal energy of the hot gases is fully transferred to the air flow. Based on the inlet and outlet temperatures of the two streams, the heat-exchanger logarithmic mean temperature difference, $\overline{\Delta T}_{\ln,\,IPH}$, is define as in (5). The required heat transfer area, A_{IPH}, can then be calculated as in equation (6) where U_{IPH} is the overall heat transfer coefficient in the IPH.

Table 1. Thermodynamic modelling: main relevant equations

System Component	*Thermodynamic Model Equations*	
Compressor 	$T_2 = T_1\left[1+\dfrac{r_C^{\left(\frac{\gamma_a-1}{\gamma_a}\right)}-1}{\eta_C}\right]$	(1)
	$\dot{m}_a = \dfrac{\dot{W}}{(1+f)\,c_{pg}\,(T_4-T_5)-c_{pa}\,(T_2-T_1)}$	(2)
Combustion Chamber 	$\dot{m}_{fuel} = \dot{m}_a\, f \Leftrightarrow \dot{m}_{fuel} = \dot{m}_a\left(\dfrac{c_{pg}\,(T_4-T_1)-c_{pa}\,(T_3-T_1)}{LHV\,0.98-c_{pg}\,(T_4-T_1)}\right)$	(3)
Turbine 	$T_5 = T_4\left(1-\eta_T\,(1-r_T^{\,(1-\gamma_g)/\gamma_g})\right)$	(4)
IPH 	$\overline{\Delta T}_{\ln,\,IPH} = \dfrac{(T_5-T_3)-(T_6-T_2)}{\ln\dfrac{(T_5-T_3)}{(T_6-T_2)}}$	(5)
	$A_{IPH} = \dfrac{\dot{m}_g\, c_{pg}\,(T_5-T_6)}{U_{IPH}\,\overline{\Delta T}_{\ln,\,IPH}}$	(6)
WHE 	$\overline{\Delta T}_{\ln,\,WHE} = \dfrac{(T_6-T_9)-(T_7-T_8)}{\ln\dfrac{(T_6-T_9)}{(T_7-T_8)}}$	(7)
	$A_{WHE} = \dfrac{\dot{m}_g\, c_{pg}\,(T_6-T_7)}{U_{WHE}\,\overline{\Delta T}_{\ln,\,WHE}}$	(8)

The hot gases are used to heat water through an external WHE. Considering a required thermal power $\dot{Q}$ to heat the water stream from T_8 to T_9, defined as, $\dot{Q} = \dot{m}_W\, c_{pW}\,(T_9 - T_8)$ where the water mass flow $\dot{m}_W$ is directly calculated and the exhaust-gases temperature, T_7, can be obtained from the adiabatic heat-balance of the heat-exchanger (elsewhere in [27]). Likewise the IPH, the logarithmic mean temperature, $\overline{\Delta T}_{\ln,\,WHE}$, and the required heat transfer area, A_{WHE} are as follows, where U_{WHE} is the overall heat transfer coefficient in the WHE and C_{pw} is the specific heat of the water. Finally, the net mechanical power delivered by the mechanical shaft of the turbine-compressor $\dot{W}$, is the difference between the mechanical powers produced by the turbine and absorbed by the compressor, $\dot{W} = \dot{W}_T - \dot{W}_C$ and the electrical power $\dot{W}_{el}$ includes the efficiency of the electrical generator, herein assumed as constant and equal to 0.93, or merely, $\dot{W}_{el} = 0.93 \cdot \dot{W}$.

3. Optimization Model

3.1. Objective Function

The objective of the present energy system is the maximisation of the Annual Worth (*AW*) of the small-scale CHP system. Therefore, the objective function is defined by the balance between the incomes and the costs from CHP system operation as described in (9):

$$\max AW = R_{sell} + C_{avoided} - C_{inv} - C_{op} \,, \tag{9}$$

where the revenues include the income from selling electricity to the grid (R_{sell}) and the avoided cost of heat generation by a conventional boiler ($C_{avoided}$). The considered costs were: the annual system investment cost (C_{inv}) and the operational costs involved in the production of electricity and heat using the CHP system (C_{op}). The annual income from selling electricity power to the grid (R_{sell}) was calculated from the cumulative amount of electricity delivered to the grid, (E_{sell}), considering the time of system operability, multiplied by the electricity-selling price, as in (10),

$$\mathrm{R_{sell}} = \mathrm{E_{sell}}\ \mathrm{p_{sell}} \,. \tag{10}$$

The electricity-selling price, (p_{sell}), was taken as a guaranteed and fixed feed-in tariff of 0.12 €/kWh, in the case presented in this study. The avoided cost $C_{avoided}$, calculated in (11), represents the avoided cost of Natural Gas that would be consumed by a conventional system (a boiler) to produce the same amount of useful thermal energy (H_{CHP}) as the CHP system.

$$\mathrm{C_{avoided}} = \mathrm{p_{fuel}}\ H_{CHP} / \eta_b \ . \tag{11}$$

The variable p_{fuel} is the Natural Gas price and η_b is the efficiency of the conventional boiler.

The annual system investment cost is calculated according to the annualised capital cost. Annualising the initial investment cost corresponds to the spreading of the initial cost across the lifetime of a system, while accounting for the time value of the money. The initial capital cost is annualised as if it were being paid off a loan at a particular interest of discount rate over the lifetime of the option. The Capital Recovery Factor (CRF) is used to determine the equal amounts of *n* cash transactions for an investment and can be expressed as in (12):

$$CRF = (P \to A, i_e, n) = \frac{i_e(1+i_e)^n}{(1+i_e)^n - 1} \,, \tag{12}$$

where *A* is the annuity (a series of equal amount cash transactions); *P* is the present value of the initial cost; i_e is the effective rate of return, and *n* is the number of years of the lifetime operation. In the present work, the lifetime of the system was limited to only 15 years. This consideration is related with the fact that, typically, micro gas turbine manufacturers recommend the replacement of the power head (it includes all the main rotating parts like the turbine and compressor rotors) after 40000 hours of operation, which corresponds to a major and expensive overhaul. In addition, some caution should be introduced when a new technology applied to an emergent market, and so, the investment risks are higher than with mature technologies and traditional markets.

For thermal-economic optimisation the effective rate of return can be approximated as: nominal rate of return (interest rate) minus inflation rate plus owners' risk factor and correction for the method of compounding [28]. The effective rate of return i_e, herein considered was 7% resulted in a CRF of 0.10979. Thus, the annual system investment cost, C_{inv}, becomes as in (13):

$$C_{inv} = \sum_i C_i\ CRF \ , \tag{13}$$

where C_i is the purchase cost of each component of the CHP system. For each component, the purchase cost equations were defined considering physical parameters that have the most effective weight at each system component (Table 2). These variables can be divided in size and quality variables. The cost estimation can be performed in order to evaluate the overall cost of the system for a specific range of application. All the considerations of cost equations are presented in [27]. The total operational costs, C_{op}, as in (14), result from the sum of maintenance and fuel costs.

$$C_{op} = p_{fuel}\ \dot{m}_{fuel}\ LHV\ t + \phi\, C_{inv} \qquad (14)$$

The fuel cost is calculated through the cumulative fuel consumption during the working periods of the CHP system (t = total number of working hours) considering the fuel price per energy unit (p_{fuel}=10 €/GJ), on a Low Heating Value (LHV) basis, and $\dot{m}_{fuel}$ is the fuel mass flow rate. In this particular study, the maintenance costs were assumed equal to 15% of the annual investment cost, roughly equivalent to 6 €/MWh$_{el}$. The salvage value of the installation is assumed to be negligible.

Table 2. Components Costs Equations

System Component	*Cost Equation*	
Air Compressor	$C_C = C_{11}\dfrac{\dot{m}_{a,\mathrm{Ref}}\left(\dfrac{\dot{m}_a}{\dot{m}_{a,\mathrm{Ref}}}\right)^{0.8} r_C\ \ln(r_C)}{0.92-\eta_c}$	(15)
Combustion Chamber	$C_{CC} = C_{21}\dfrac{\dot{m}_{a,\mathrm{Ref}}\left(\dfrac{\dot{m}_a}{\dot{m}_{a,\mathrm{Ref}}}\right)^{0.8}\left(1+e^{C_{22}(T_4-C_{23})}\right)}{0.995-{P_4}/{P_3}}$	(16)
Turbine	$C_T = C_{31}\dfrac{\dot{m}_{g,\mathrm{Ref}}\left(\dfrac{\dot{m}_g}{\dot{m}_{g,\mathrm{Ref}}}\right)^{0.8}\left(1+e^{C_{32}(T_4-C_{33})}\right)\ln(r_T)}{0.92-\eta_T}$	(17)
Internal Pre-Heater	$C_{IPH} = C_{41}A_{IPH}^{\ 0.4}\left(1+e^{C_{42}(T_5-C_{43})}\right)$	(18)
Water Heat-exchanger	$C_{WHE} = C_{51}.A_{WHE}^{\ 0.4}$ o	(19)

3.2. Decision Variables

Six decision variables were selected for the optimisation algorithm. The chosen decision variables are: the compressor ratio (r_C), the air compressor isentropic efficiency (η_C), the isentropic efficiency of the turbine (η_T), the air temperature at the internal pre-heater exit (T_3), the temperature of the combustion gases at the turbine inlet (T_4) and the electrical power production ($\dot{W}$). All the decision variables were lower and upper bounded: $3.0 \le r_C \le 6.0$; $0.70 \le \eta_C \le 0.90$; $0.70 \le \eta_T \le 0.90$; $500 \le T_3 \le 1000$; $1000 \le T_4 \le 1400$ and $90 \le \dot{W} \le 120$.

3.3. Non-linear Constraints

Physical limitations of system operation were also formulated in terms of nonlinear inequality constraints. The definition of these constraints aims to restrict the implicit variables of the problem according to their physical significance in the system operation. As it is well known, the internal pre-heater uses the heat in the exhaust gas to boost the compressed air temperature, prior to combustion. This increase in the combustion inlet temperature leads to a reduction in the amount of fuel that is required, and thus, the efficiency of the system is improved. The high-pressure air is pre-heated before entering in the CC and so it is required that the temperatures T_2 should be lower than the temperature T_3. On the other hand, T_3 must be lower than the exhaust gases temperature (T_5) at the turbine exit, in order to guarantee an effective heat transfer in the IPH. This physical limitation can be satisfied by the constraint: $T_2 \le T_3 \le T_5$. Similarly, the inlet temperature of the air (T_1) must be lower than the air temperature after suffering the pre-heating in the IPH (T_3) and, therefore, the latter must be lower than the temperature of the combustion gases at the turbine inlet (T_4) which is the highest temperature reached in the system, $T_1 \le T_3 \le T_4$. Also, the difference between the inlet and outlet temperatures in each of heat exchangers flows should be bounded to ensure the effectiveness in the process of heat transfer between the fluids: $50 \le T_3 - T_2 \le 600$, $10 \le T_5 - T_3 \le 700$, $10 \le T_6 - T_2 \le 800$, $200 \le T_6 - T_9 \le 800$. The temperature T_7 cannot be lower than 363K in order to prevent the condensation problems in the heat recovery system $363 \le T_7 \le 1000$. Primary Energy Savings (PES) was also include in the model as the eighteenth inequality constraint in order to guarantee that the system may be classified as high-efficient CHP power plant, which corresponds to a value above 10% for small-scale applications ($PES > 0.10$).

4. Optimization Method: GA

A genetic algorithm is a well-known technique that performs space search through natural selection principles. The algorithm is built upon the definition of a population and through successive operations that mimic natural selection, such as gene exchange and gene mutation. Through this process the population evolves such that the fittest individuals survive and therefore give rise to new and stronger individuals. The success of these algorithms lays on the effective design of a number of operations common to genetic algorithms, i.e. cross-over and mutation operators, mutation rate, elitist strategy and convergence criteria. The choice of these parameters has a great influence on the speed of convergence as well as on the success of the optimization. For this purpose a genetic algorithm, based in Java programming language, was developed to better accommodate the specificities of the problem and allow full control of the search problem. Evolution starts with a random population of individuals that is defined by a floating point representation of the decision variables presented earlier along with the calculated fitness value. Each individual in the population is ranked according to their fitness. The initial population, generated randomly, evolves based on the selection, crossover and mutation operators with the objective of maximizing the annual worth of the small scale-CHP system.

The implementation of this genetic algorithm follows most of the classical approaches [29,30] though some adjustments have been introduced that proved better considering the attained results. Offspring generation and selection is performed in a continuous way so that population size is maintained identical at all times. This way the population is continuously updated to reflect its evolution instead of following the standard generation cycle where the population increases, when producing offspring's, and decreases, when natural selection occurs. Cross-over is performed through the exchange of genes, i.e. decision variables – single genes are randomly selected and exchanged when performing cross-over giving rise to a more stable population preventing rapid disruption from established valid individuals as well as preserving population diversity. Nevertheless there is a 15% rejection rate of offspring's given their unfitness pertaining to the nonlinear inequality constraints, i.e. thermodynamic constrains.

Constraint handling is hardcoded in the implementation along with the calculations pertaining to the objective function providing early stage unfitness detection which liberates computer power and augments the searching performance - yielding a better computational performance since thermodynamic constraints are progressively tested giving rise to new candidates if fully confirmed, otherwise promptly rejected if during calculations any of the constraints is not verified.

In line with the previous argument it was allowed for single gene mutations occurring with probability *p*. Best results are attained with a small number of iterations and is better achieved with a population size higher than 50, Fig. 2-a). Mutation probability has been found to have little interference concerning population diversity, Fig2-b). Keeping the mutation probability low, less than 0.01, proves to perform slightly better and lead to faster conversion rates and better solutions.

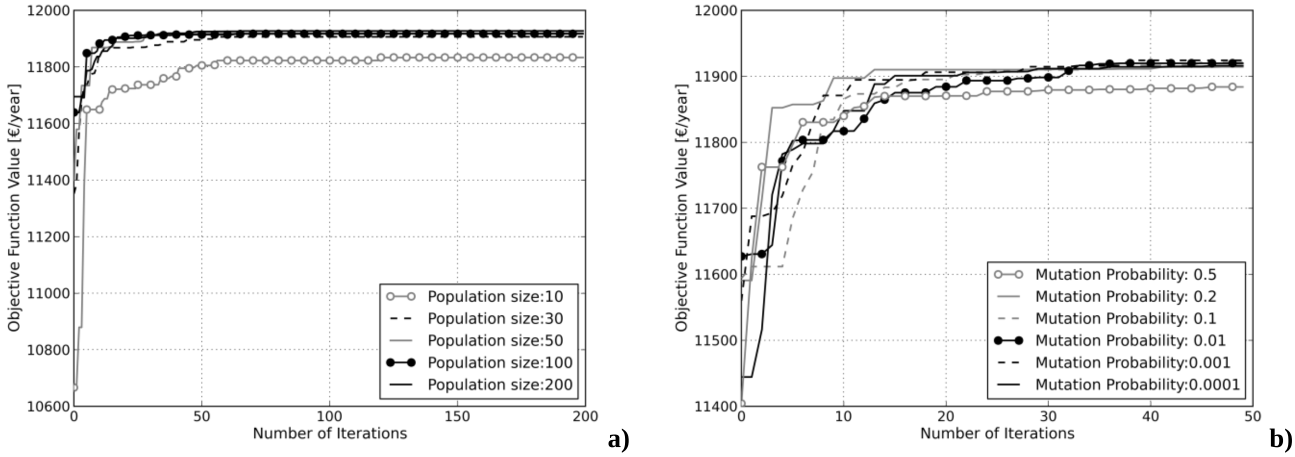


*Fig. 2. GA performance analysis: **a)** Evaluation of objective function vs iterations number for different population sizes (mutation probability=0.1 & 200 iterations); **b)** Evaluation of objective function vs iterations number for different mutation probability values (population size=100 & 50 iterations).*

5. Results and Discussion

The cogeneration model described was tested for a fixed thermal power production equal to 125kW, which was assumed as the base thermal load of the building. A base case scenario was simulated considering the natural gas with a LHV of 45100kJ/kg, the electricity-selling price of 0.12€/kWh and the fuel price of 10€/GJ. When the optimisation method reaches the convergence, the optimal values for the decision variables and costs are obtained. After convergence was achieved, the optimal economic values are listed the Table 3. Results shows that for a base case scenario it is possible to obtain an annual revenue of 46204.4 €/year if all the electricity generated is sold to the grid. Arising from the fact that there is no need to have a separate system to produce the total heating demand, the economic benefit from that avoided cost was also accounted and it represents a revenue of 20 000 €/year considering a typically boiler running with NG and with a thermal efficiency of 90%. Concerning the costs, the predominant economic charge is the operational cost which includes the fuel costs as well as the maintenance costs, 41 287 €/year. The optimal cogeneration system would represent a total initial investment cost of 118 329 €, which annualized over a 15 year's period of estimated lifetime represents almost 24% in terms of total annual costs. According to the results it is possible to obtain a positive profit from system operation. The maximum annual worth is of 11 925 €/year.

Table 3. Optimal annual worth and operating costs of the small CHP system

Objective Function and respective values	*Optimal Value, €/year*
Capital investment Cost, C_{inv}	12 992.9
Total Operating Costs C_{op}	41 287.3
Income from selling electricity power to the grid, R_{sell}	46 204.4
Avoided cost of conventional heat generation $C_{avoided}$	20 000.0
Annual worth of the small scale-CHP system	**11 925.2**

This economic output corresponds to the optimal values obtained for the six decision variables as shown in Table 4. The optimal solution depicts a micro-gas turbine with a compressor pressure ratio of $r_C =$ 5.831 and a turbine inlet temperature of 1381 K. These two decision variables, together with the regenerator effectiveness, are the most important parameters in micro gas turbines cycles. Both results are higher than the values for the models currently available in the market (an r_C of 4 and a T_4 of approximately 1 200 K). A possible explanation for this difference is the use of low cost materials in the equipment manufacturing, which imposes some limitations to these operational variables. The need to increase combustion temperature to increase the efficiency of gas turbines has resulted in a large increase in Turbine Inlet Temperature (TIT), but this increase has been accompanied by several changes in the materials for several components (Ni-base cast super alloys), and consequently, increased manufacturing costs. The compressor and turbine isentropic efficiencies (83.6% and 86.9%, respectively) seem to be within the expected values for this kind of systems. According to the results for the optimal solution, the resulting CHP system is able to produce about 103.5 kW of electrical power. Considering the electricity production output, the optimal system has a heat-to-power ratio of $\lambda = 1.21$. In Table 4, the results of efficiencies and performance criteria are also depicted.

Table 4. Optimal results for the decision variables and most important parameters of the CHP system

	System Component		*Optimal Value*
Optimal Six Decision Variables	Compressor	r_C	5.831
		η_C (%)	83.64
	Combustion Chamber	T_3 (K)	974.3
	Turbine	T_4 (K)	1381
		η_T (%)	86.87
	Electrical Power	$\dot{W}$ (kW_{el})	103.5
Performance Criteria	Internal Pre-heater	ε_{IPH} (%)	96.28
	Water Heat-exchanger	ε_{WH} (%)	76.12
	Electrical efficiency	η_{el} (%)	35.26
	Overall Efficiency	$\eta_{overall}$ (%)	80.99
	Primary Energy Savings	PES (%)	15.22

Considering the results for the various operational variables, an electrical efficiency of 35.26% was obtained that is relatively higher than the current values observed with the real micro-gas turbines 25–31%). The total efficiency of 80.99% is a reasonable value for the use of micro turbines on cogeneration applications. The performance of a cogeneration system can be evaluated by comparison with the separate production of heat and electricity. In this study, PES were calculated considering the guidelines that established harmonized efficiency reference values for separate production in application of Directive 2004/8/EC. The calculations included the correction factors regarding the average local climate and the avoided grid losses. The optimal configuration allows a PES of 15.22%, revealing that the optimal solution corresponds to a high efficient cogeneration system and that respects the guidelines in the legislation.

6. Conclusions

A nonlinear optimisation model was developed for determining the optimal small-scale CHP system for a given thermal demand, 125 kW_{th}. This latter technological optimization includes all the relevant internal components and introduces complexity to the model, via the increased number of decision variables. So, an efficient numerical method is required to solve this constrained non-linear optimization problem. Genetic algorithms are well known in finding optimal solutions in complex high-dimensional, multimodal problems and was hence designed and implemented to fit the non-linear problem hereby presented. The algorithm differs from the standard implementations in that it performs continuous population updates regardless of generation cycle completion. Also, single gene cross-over operations have been used to reduce the number of unfit offspring. The algorithm, after a small number of iterations, was able to specify a cogeneration system with a positive annual profit (i.e. 11 925 €/year). This economic output was obtained for a micro-gas turbine able to produce 103.5 kW of electrical power in cogeneration mode. The combination of the optimal decision variables disclose a system with high performance output: the IPH as an effectiveness above 96%, the electrical efficiency is higher than the commercial models within the same energy output and it allows a positive primary energy saving index of about 15%. In brief, the optimal sizing of micro-gas turbines has a dynamic role in their overall economic viability: decreasing the size typically increases the specific cost of the equipment. As future work, it is intended to compare the results from this genetic algorithm in development with the results from other optimization algorithms, such as: the Sequential Quadratic Programming (SQP), one of the most successful methods for the numerical solution of constrained nonlinear optimization problems; Pattern Search MADSPositiveBasis2N, both derivative free optimization methods.

Nomenclature

A	*Heat transfer area, m^2*
AW	*Annual Worth, €*
C	*Cost, €*
CRF	*Capital Recovery Factor, (-)*
c_p	*Specific heat, kJ/kg K*
f	*Mass fuel to air ratio, (-)*
i	*Rate of Return, %*
LHV	*Low Heating Value, kJ/kg*
$\dot{m}$	*Mass flow rate, kg/s*
n	*Nº of operation years, (-)*
NG	*Natural Gas, (-)*
P	*Pressure, bar*
PES	*Primary Energy Saving, (%)*
p	*Price, €*
r	*Compression ratio, (-)*
t	*Time, h*
T	*Temperature, K*
U	*Global heat transfer, W/m^2K*
$\dot{W}$	*Net mechanical power, kW*

Greek symbols

ε	*Heat exchanger effectiveness*
γ	*Polytrophic adiabatic coefficient*
η	*Efficiency*
φ	*Rate of operation and maintenance*

Subscripts and superscripts

avoided	Avoided cost
C	Compressor
CC	Combustion chamber
i	CHP system component
inv	Annual system investment costs
e	Effective
el	Electrical
op	Operational costs
g	Gases
IPH	Internal Pre-Heater
sell	Selling electricity revenue/price
T	Turbine
WHE	Water Heater Exchanger

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