# The analytical method to obtain the optimal phase change temperature in BCHP system with the consideration of limited storage period

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#### Abstract:

Thermal Energy Storage (TES) is an advanced energy option to balance the energy demand and supply in Building Cooling, Heating and Power (BCHP) systems, for the purpose of achieving higher system energy efficiency. The constraint of limited storage period was not considered in the past research, which is critical that influences the selection of phase change temperature and the system performance.

To maximize the performance of BCHP associated with a latent TES and an absorption heat pump, the analytical method of maximizing the non-dimensional cooling capacity is put forward to achieve the optimal specific phase change temperature in conjunction with a specific storage period. With the consideration of limited storage period, limited NTU and limited coefficient of thermal conductivity are put into the mathematical TES model. Based on that the analytical mathematical model with 9 variables and 8 equations, the optimal phase change temperature of TES is obtained. The results show that the optimal phase change temperature is higher than the value achieved by the current research.

#### Keywords:

BCHP; LTES; optimal phase change temperature; NTU; limited storage period

### 1. Introduction

Thermal Energy Storage (TES) is an advanced energy option to balance the energy demand and supply in Building Cooling, Heating and Power (BCHP) systems, for the purpose of achieving higher system energy efficiency. Among all kinds of energy storage methods, latent thermal energy storage (LTES) is attractive because of its high energy density and near-isothermal operation. Selecting the thermal properties, especially phase change temperature, is vital for the high thermal performance of BCHP with TES.

The Early studies concluded that the best thermal performance was achieved when the phase change temperature is equal to the geometric average of the environment temperature and the temperature of the hot HTF during charging <sup>[1~3]</sup>, with considering effect of natural convection during PCM melting. Bellecci and Conti <sup>[4]</sup> considered LTES for solar power generation, the HTF leaving the LTES during charging was delivered to the heat engine. The optimal melting temperature was found to be approximately the arithmetic mean of the inlet temperatures of the hot and cold HTF. The finite duration of the discharge process was not taken into consideration in any of preceding studies except for Bellecci and Conti <sup>[4]</sup> who assumed a fixed discarging duration. Hamidreza Shabgard <sup>[9]</sup> took into account the practical constraint of the storage period in studying LTES for the solar power generation. And the optimal phase change temperature was found corresponding to charging inlet temperature.

In related studies, the researchers have done a lot of work on the performance enhancement of

BCHP by incorporating TES. Chunhao Huang (2008) <sup>[5]</sup> applied the ideal TES model, which is with high enough storage capacity and charging/discharging rate, to evaluate the thermal performance in different kinds of buildings, the water storage with given volume and maximum charging/discharging rate was studied in Yonghong Li (2005) <sup>[6]</sup> and Aiguo Liu (2010) <sup>[7]</sup>'s research. Celador (2011) <sup>[8]</sup> introduced three different models for hot water storage: the ideal model, the actual model and the mixed model, in order to achieve the specific prime mover capacity in BCHP systems.

All in all, the research remains blank for the optimal phase change temperature of LTES for BCHP systems. The objective of this study is to investigate the optimal phase change temperature of an LTES system for BCHP accounting for the finite duration of a charging/discharging cycle. Based on the previous achievements, the heat transfer analysis is developed to quantify the performance of a LTES for BCHP, taking into account the constraint of limited storage period.

## 2. System Configuration

BCHP systems are composed of a gas turbine, a LTES device and an absorption heat pump, operating in grid parallel mode. As shown in Fig.1, the LTES was placed between the gas turbine and the absorption heat pump. The gas turbine generates electricity, while the LTES and thermal load supplied by the absorption heat pump. The excess electricity can be sold back to the power grid. When the heating/cooling load is lower than the average, a portion of exhaust gas is diverted to the LTES during the charging process. When the heating/cooling load is higher than the average, the heat is diverted from the LTES to the absorption heat pump during the discharging process.



Fig. 1. The energy generation plant: Brayton tri-generation cycle with TES

## 3. Mathematical model

One-dimensional heat transfer is assumed throughout the entire process. Of interest is the performance of LTES taking into account the practical constraint: a entire charging-discharging cycle takes no more than 24 hours. The assumptions for the theoretical model are as follows:

- 1) LTES has sufficient storage capacity
- 2) The properties of the exhaust gas and PCM are assumed to be constant.
- 3) The PCM is a pure material with a unique phase change temperature.
- 4) The sensible heat transfer is taken to be negligible (the Stefan number is zero).

5) The heat loss from the LTES channel is negligible.

Based on those assumptions, the performance of BCHP with LTES is defined by the nondimensional cooling capacity within given storage period <sup>[10]</sup>.

$$\overline{Q_{eff}} = \frac{Q_{eff}}{(mc_f)_c T_0 t_{cyc}}$$
(1)

Where,  $Q_{eff}$  is the cooling capacity within the storage period , kJ;  $(\stackrel{"}{mc_f})_c$ , the thermal mass flow of HTF in charging process, J/(K s);  $T_0$ , the surrounding temperature, K; t<sub>cyc</sub>, the storage period, s.

#### 3.1. Charging process

The analysis of charging process builds upon the conduction model, as shown in Fig.2 in Refs<sup>[2, 3]</sup>. The governing equations are,



Fig. 2. the LTES during charging

$$\binom{\square}{mc_f}_c (T_{c,i} - T_{c,o}) = h_{f,c} L_f \frac{T_{c,i} - T_{c,o}}{\ln(\frac{T_{c,i} - T_{c,s}}{T_{c,o} - T_{c,s}})}$$
(2)

$$\begin{pmatrix} \Box \\ mc_f \end{pmatrix}_c (T_{c,i} - T_{c,o}) = \lambda_p L_p \frac{T_{c,s} - T_p}{\delta_c}$$
(3)

$$\begin{pmatrix} {}^{\Box}\\ m c_f \end{pmatrix}_c (T_{c,i} - T_{c,o}) = \rho_p r_p L_p \frac{d\delta_c}{dt}$$
(4)

Where,

 $T_{c,s}$  —the channel wall temperature in charging process,K;

 $h_{f,c}$ —the forced convection heat transfer coefficient associated with the exhaust gas in the channel in charging process,  $W/(m^2K)$ ;

 $\lambda_p$ —the thermal conductivity of PCM, W/(mK);

 $r_p$ —the latent heat of PCM, J/kg;

 $L_f$ —the surface area of the channel wall on the exhaust gas side,  $m^2$ ;

 $T_p$ —phase change temperature, K;

 $L_p$ —the surface area of the channel wall on PCM side,  $m^2$ ;

 $\delta_{c}$ —the melting thickness, m.

The non-dimensional solution of the above equations are,

$$\bar{T_{c,s}} = \frac{T_{c,s}}{T_0} = \frac{T_{c,i}/f_c + T_p/\sigma_c}{1/f_c + 1/\sigma_c}$$
(5)

$$\bar{T_{c,o}} = \frac{T_{c,o}}{T_0} = \bar{T_{c,s}} + (\bar{T_{c,i}} - \bar{T_{c,s}}) \exp(-NTU_c)$$
(6)

$$\sigma_c(\tau_c) = -f_c + \sqrt{f_c^2 + 2(T_{c,i} - T_p)\tau_c}$$
<sup>(7)</sup>

Where,

 $\sigma_c$  —the non-dimensional melt thickness in charging process, defined by (8);

 $A_{fp}$ —the surface area ratio defined by (9);

 $NTU_c$ —the number of heat transfer units in charging process, defined by (10);

 $\tau_c$ —the non-dimensional charging time, defined by (11);

 $f_c$ —the non-dimensional constant in charging process, , defined by (12);

$$\sigma_c = \frac{h_{f,c} \delta_c}{\lambda_p} \tag{8}$$

$$A_{fp} = L_f / L_p \tag{9}$$

$$NTU_c = \frac{h_{f,c}L_f}{(mc_f)_c}$$
(10)

$$\tau_c = \frac{h_{f,c}^2 T_0}{\rho_p \lambda_p r_p} t_c \tag{11}$$

$$f_c = \frac{A_{fp} NTU_c}{1 - \exp(-NTU_c)} \tag{12}$$

### 3.2. Discharging process

The analysis of charging process builds upon the conduction model, as shown in Fig.3 in Refs [2, 3]. The governing equations are.



Fig. 3. the LTES during discharging

$$\begin{pmatrix} \Box \\ mc_{f} \end{pmatrix}_{d} (T_{d,o} - T_{d,i}) = h_{f,d} L_{f} \frac{T_{d,o} - T_{d,i}}{\ln(\frac{T_{d,s} - T_{d,i}}{T_{d,s} - T_{d,o}})}$$
(13)

$$\begin{pmatrix} \Box \\ mc_f \end{pmatrix}_d (T_{d,o} - T_{d,i}) = \lambda_p L_p \frac{T_p - T_{d,s}}{\delta_d}$$
(14)

$$\begin{pmatrix} \Box \\ m c_f \end{pmatrix}_d (T_{d,o} - T_{d,i}) = \rho_p r_p L_p \frac{d\delta_d}{dt}$$
(15)

Where,

 $T_{d,s}$ —the channel wall temperature in discharging process, K;

 $h_{f,d}$ —the forced convection heat transfer coefficient associated with the exhaust gas in the channel in discharging process,  $W/(m^2K)$ ;

 $\delta_d$ —the solidifying thickness, m.

The non-dimensional solution of the above equations are,

$$\bar{T_{d,s}} = \frac{T_{d,s}}{T_0} = \frac{\bar{T_{d,i}} / f_d + \bar{T_p} / \sigma_d}{1 / f_d + 1 / \sigma_d}$$
(16)

$$\bar{T}_{d,o} = \frac{T_{d,o}}{T_0} = \bar{T}_{d,s} - (\bar{T}_{d,s} - \bar{T}_{d,i}) \exp(-NTU_d)$$
(17)

$$\sigma_{d}(\tau_{d}) = -f_{d} + \sqrt{f_{d}^{2} + 2(\bar{T_{p}} - \bar{T_{d,i}})\tau_{d}}$$
(18)

Where,

 $\sigma_d$ —the non-dimensional melt thickness in discharging process, defined by (19);  $NTU_d$ —the number of heat transfer units in discharging process, defined by (20);  $\tau_d$ —the non-dimensional discharging time, defined by (21);  $f_d$ —the non-dimensional constant in discharging process, defined by (22).

$$\sigma_d = \frac{h_{f,d}\delta_d}{\lambda_p} \tag{19}$$

$$NTU_{d} = \frac{h_{f,d}L_{f}}{(mc_{f})_{d}}$$
(20)

$$\tau_d = \frac{h_{f,d}^2 T_0}{\rho_p \lambda_p r_p} t_d \tag{21}$$

$$f_d = \frac{A_{fp} NTU_d}{1 - \exp(-NTU_d)}$$
(22)

### 3.3. Coupled charging-discharging process

The practical constraint of the storage limited period is shown in (23).

$$t_{c,\max} + t_{d,\max} = t_{cyc} \tag{23}$$

Where,

 $t_{c,\max}$ —the charging time, s;

 $t_{d,\max}$ —the discharging time, s;

 $t_{cyc}$ —the total storage period, *s*.

The equation can be non-dimensionlized to be,

$$\tau_{cyc} = \frac{h_{fd}^2 T_0 t_{cyc}}{\rho_p \lambda_p r_p} = \tau_{c,\max} + h_{cd}^{-2} \tau_{d,\max}$$
(24)

Energy stored is equal to energy extracted in LTES can be expressed as,

$$\delta_{c,\min} = \delta_{d,\min} = 0; \delta_{c,\max} = \delta_{d,\max}$$
(25)

Which is,

$$\sigma_{c,\max} = h_{cd} \, \sigma_{d,\max} \tag{26}$$

Where,

 $\tau_{c,\max}$ —the non-dimensional charging time, defined by (27);

 $\tau_{d,\max}$ —the non-dimensional discharging time, defined by (28);

 $\bar{h_{cd}}$  —the convection heat transfer coefficient ratio of charging to discharging, defined by (29);  $\sigma_{c,max}$ —the maximum melting thickness;

 $\sigma_{d,\max}$ —the maximum solidifying thickness.

$$\tau_{c,\max} = \frac{h_{f,c}^2 T_0}{\rho_p \lambda_p r_p} t_{c,\max}$$
(27)

$$\tau_{d,\max} = \frac{h_{f,d}^2 T_0}{\rho_p \lambda_p r_p} t_{d,\max}$$
(28)

$$\bar{h_{cd}} = \frac{h_{fc}}{h_{fd}} \tag{29}$$

### 3.4. The thermal performance of BCHP

For BCHP shown in Fig.1, the total cooling capacity in one storage period of BCHP system can be written as,

$$Q_{eff} = \int_0^{t_{d,\text{max}}} P_d(t_d) \Box COP(t_d) dt_d$$
(30)

Where,

$$P_{d}(t_{d}) = (\overset{\cup}{m}c_{f})_{d}(T_{d,o}(t_{d}) - T_{d,i})$$
(31)

$$COP(t_{d}) = X(1 - \frac{T_{c}}{T_{d,o}(t_{d})}) \frac{T_{e}}{T_{c} - T_{e}} = X(1 - \frac{T_{c}}{T_{d,o}(t_{d})}) \frac{T_{e}}{\bar{T_{c}} - \bar{T_{e}}}$$
(32)

Where,

 $t_c$ —the time point in charging process, s;

 $t_d$ —the time point in discharging process, s;

 $T_{d,a}^{-}$ —the non-dimensional outlet temperature in discharging process, defined by ,

$$\bar{T_{d,o}} = \frac{T_{d,o}}{T_0}$$
(33)

From the above equations, ten design operating parameters contribute to the thermal perform of BCHP are  $T_{c,i}$ ,  $T_p$ ,  $T_{d,i}$ ,  $NTU_c$ ,  $NTU_d$ ,  $\tau_{c,\max}$ ,  $\tau_{d,\max}$ ,  $A_{fp}$ ,  $h_{cd}$ ,  $h_{f,d}$ . Hence, prediction of BCHP thermal performance corresponding to the phase change temperature involves the specification of nine of the ten variables listed above.

### 4. Results and discussion

The boundary conditions and initial conditions are based on the practical system, as listed in Table 1. In general, by maximizing the total cooling capacity in one storage period, the optimal phase change temperature can be achieved, as shown in Fig.4.

	Parameter	Symbol	Value	Unit
Phase change material	Density	$ ho_p$	800	kg/m <sup>3</sup>
	Thermal Conductivity	$\lambda_{_p}$	0.7	$W/m \cdot K$
	Latent Heat	$r_p$	100	kJ/kg
Exhaust gas	Volume Flow	G	15	$m^3/s$
	Density	$ ho_{f}$	0.614	kg/m <sup>3</sup>
	Specific heat capacity	$\mathcal{C}_{f}$	1.145	$J/(kg \cdot K)$
Absorption heat pump	Thermodynamic perfect degree	Х	0.2	-
	Vapor temperature	T <sub>e</sub>	5	${}^{\mathcal{C}}$
	generate/condense temperature	$T_{c}$	35	${}^{\mathcal{C}}$
Design parameters	the number of heat transfer units in charging process	NTU <sub>c</sub>	10	-
	the number of heat transfer units in discharging process	NTU <sub>d</sub>	10	-
	the surface area of the channel wall on the exhaust gas side	L <sub>f</sub>	10	т
	the surface area of the channel wall on the phase change material side	L <sub>p</sub>	10	т
Operating parameters	Inlet temperature in charging process	$T_{c,i}$	550	$^{\circ}\!C$
	Inlet temperature in discharging process	$T_{d,i}$	170	$^{\circ}\!C$
	The convection heat transfer coefficient in charging process	$h_{fc}$	60	$W/(m^2 \cdot C)$
	The convection heat transfer coefficient in discharging process	$\mathbf{h}_{\mathrm{fd}}$	60	$W/(m^2 \cdot C)$
	The storage period	$t_{cyc}$	24	h

Table 1. the given parameters of BCHP



Fig. 4. Variations of the stored heat and non-dimensional cooling capacity with the phase-change temperature in one storage period

As shown in Figure 4, an optimal value of phase-change temperature exists corresponding to the maximum amount of the total stored heat. This is because, with the increase of the phase change temperature, the charging energy flow decreases as temperature different decreases, while the discharging energy flow increases. Since in the complete cycle, the heated stored in the charging process is equal to the heat released in the discharging process, the maximum amount of the total stored heat is achieved at the optimal phase-change temperature which is the geometric mean value of the charging and discharing inlet temperatures, as been proven in the previous research.

There also exists an optimal value of phase-change temperature corresponding to the maximum amount of the total cooling capacity. The optimal phase-change temperature for the maximum cooling capacity is higher than the one for the maximum stored heat. That is because that the non-dimensional cooling capacity is related to the total stored heat and the absorption heat pump COP, while COP increases with the increase of the phase-change temperature.

For the given boundary and initial conditions, the optimal phase-change temperature is approximately 390 °C. The total stored heat is related to the heat flow rate and the corresponding charging time. As shown in Figure 5, the charging time linearly increases with the increase of the phase-change temperature. Apparently, the average charging heat flow rate is increasing while the average discharging heat flow rate decreases with the increase of the phase-change temperature. Thus, there exists an optimal phase-change temperature which is approximately  $360^{\circ}$ C.



Fig. 5. The variations of charging/discharging time with the phase-change temperature

## 5. Conclusions

The analytical method of maximizing the non-dimensional cooling capacity is put forward to achieve the optimal specific phase change temperature in BCHP, with the consideration of limited storage period. And the conclusions are obtained as follows,

(1) Limited NTU and limited coefficient of thermal conductivity are also put into the mathematical TES model.

(2) Based on the analytical mathematical model with 9 variables and 8 equations, the optimal phase change temperature of TES is obtained.

(3) The results show that the optimal phase change temperature drifts to the higher temperature by considering the constraints than the traditional method.

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