Optimal speed control of two unequal parallel pumps in reservoir filling: minimum energy with fixed time

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Abstract:

Pumping systems are responsible for large energy consumption in the industrial sector. A considerable amount of energy can be saved by changing pump rotational speed using Variable Speed Drives (VSD) when necessary. As an example, we present how VSDs can be applied to a pumping process in which two unequally sized parallel pumps are used to pump a certain fluid volume from one tank to another. The rotational speeds of the pumps are controlled simultaneously and independently according to the changing liquid levels in the tanks. The objective is to complete pumping with minimum energy consumption and in fixed time. We derive an optimal control law for rotational speeds using the Calculus of Variations and discuss the implementation of this law in actual control systems.

Keywords:

Parallel pumping, Reservoir filling, Variable speed drives, Optimal control, Calculus of variations.

1. Introduction

Operating pumps, fans, and compressors constitutes almost 20 % of the world's electricity use, and in some industries even 50 % of electricity is consumed in these applications [1]. The possibility to save energy by improving the efficiency of the equipment is limited since the total efficiencies of the best pumps are already as high as 95 % [2]. However applying variable speed drives (VSD) in pumping systems has now been identified as the most effective way to save energy [3].

In reservoir pumping applications, a given amount of liquid is pumped from one reservoir to another, for example, in filling and emptying a water storage tank and pumping waste water. In reservoir filling, optimal pump rotational speed control has been studied both analytically and experimentally. Bene and Hős [4] derived an analytical expression for rotational speed yielding minimum specific energy under certain simplifying assumptions about the shape of pump characteristic curves. Tamminen et al. [5] presented an implementation of minimum specific energy control using a programmable VSD, and later on Ahonen [6] determined experimentally the time and energy consumption of the reservoir filling process for different fixed rotational speeds. The drawback of considering only pumping energy is that rotational speed becomes low and pumping time long. In practice, the pumping time may be limited. Karassik et al. [7], p. 11.15, suggested that a constant flow rate could be used in reservoir pumping applications, which is clearly correct if the surface levels do not change during the process. In an earlier study [8], we considered minimizing energy consumption in this process when process time is fixed and showed that maintaining a constant flow rate is indeed a good compromise solution for energy and time. Besides, pump reliability remains high.

Parallel pumps are used to manage varying flow rates, for example, in waste water pumping stations [9]. Redundant parallel pumps increase the flexibility and reliability of the process because one pump can easily be disengaged for maintenance. Various control schemes and optimization criteria have been used to optimize parallel pumping systems. For instance, two parallel pumps can be controlled so that one is driven at full speed whereas the rotational speed of the other is varied [3],[7]. To achieve energy efficient operation, da Costa Bortoni et al. [10] minimized the departure from the BEP in a two-pump system. Viholainen et al. [11] showed that energy consumption can be minimized by controlling the pumps simultaneously and independently. As with a single pump, control procedures of parallel pumps that focus only on energy result in low rotational speeds and long process times. When a time constraint is included, optimal control procedure is not obvious and must be found through mathematical analysis.

The optimal control law derived for a single pump in reservoir filling [8] applies to equally sized parallel pumps when flow rate and power are divided by the number of pumps. With two unequal parallel pumps, the control procedure becomes more complicated because of more degrees of freedom. We present a simple mathematical result for optimal rotational speeds in this kind of simultaneous control. Our method does not yet take into account the possibility of shutting down either pump.

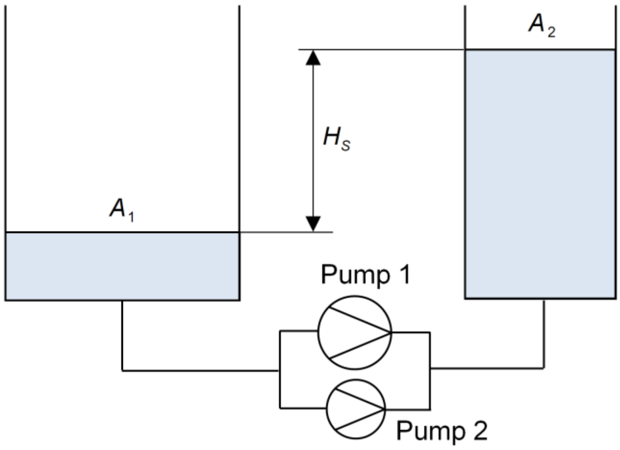

Fig. 1. Parallel pumping between reservoirs. Reservoir shapes and sizes can be arbitrary.

2. Pump and system hydraulics in parallel pumping

To assess the energy savings achieved by lowering pump rotational speed, we must know the effect of this change on pump performance. Pump head and efficiency curves at the rotational speed n can be obtained from the curves that are known at some reference rotational speed n_r. The change in pump head (pressure rise divided by ρg) and power when rotational speed and flow rate are changed can be described using the following affinity laws:

$$\frac{Q}{Q_r} = \frac{n}{n_r} \qquad (1)$$

$$\frac{H(Q)}{H_r(Q_r)} = \left(\frac{n}{n_r}\right)^2 \qquad (2)$$

$$\frac{P(Q)}{P_r(Q_r)} = \left(\frac{n}{n_r}\right)^3 \qquad (3)$$

where $H_r(Q_r)$ and $P_r(Q_r)$ are the known head and power curves at the reference rotational speed n_r, which is usually the maximum allowed rotational speed. According to above affinity laws, pump power is approximately proportional to the third power of rotational speed, which motivates lowering rotational speed with a VSD when possible. According to (1)-(3), the pump efficiency $\eta = \rho gQH/P$ does not depend on the rotational speed. However, in reality pump efficiency drops when rotational speed is reduced, which can be taken into account using the following equation [12]:

$$\frac{1-\eta(Q)}{1-\eta_r(Q_r)} = \left(\frac{n_r}{n}\right)^k \quad (4)$$

The pump operates at the intersection of pump and system head curves. The system curve is composed of a static head (H_s), which does not depend on flow rate, and a frictional head, which does. For simplicity, we assume that frictional head is quadratic with respect to flow rate:

$$H = H_s + KQ^2 \quad (5)$$

We study a process with two parallel pumps, whose characteristic head curves, which correspond to reference speeds n_{1r} and n_{2r}, are shown in Fig. 2. When several pumps operate in parallel, every pump has the same head, and their flow rates are additive. However, a pump in parallel can operate only when system head is lower than shut-off head. This means that if the former exceeds the latter of pump 2, $H_{2,SO}$, pump 2 must be shut down to prevent backflow and pump failure. When static head is $H_{s,min}$, flow rate through the example system is $Q=Q_1+Q_2$, which depends on the rotational speeds of pumps 1 and 2:

$$Q(H) = H_1^{-1}(H, n_1) + H_2^{-1}(H, n_2) \quad (6)$$

where $H_1(Q,n_1)$ and $H_2(Q,n_2)$ are pump head curves from (2). The reference head curves H_{1r} and H_{2r} are given later in Section 5. The operating point (Q,H) of two parallel pumps at given rotational speeds of n_1 and n_2 can be solved iteratively from (1)-(2) and (5)-(6) when the system head H is given.

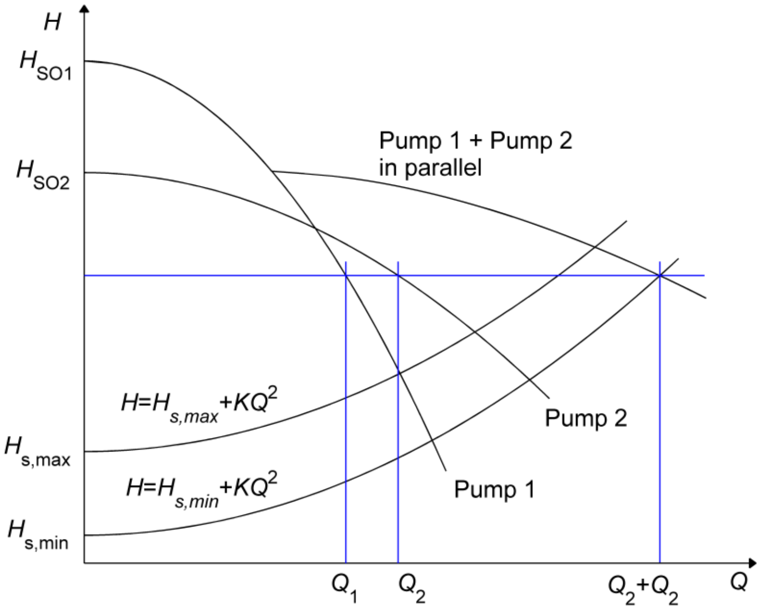

Fig. 2. Operating point of a parallel pumping system and two system curves corresponding to low ($H_{s,min}$) and high ($H_{s,max}$) static heads.

The total power of two pumps in parallel depends on the operating point:

$$P = \rho g H\left(\frac{Q_1}{\eta_1} + \frac{Q_2}{\eta_2}\right) \quad (7)$$

The time and energy consumption of a pumping process can be obtained by integration over the process

$$E = \int_{H_{s,\min}}^{H_{s,\max}} \frac{P}{Q}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)^{-1} \mathrm{d}H_s \quad (8)$$

$$T = \int_{H_{s,\min}}^{H_{s,\max}} \frac{1}{Q}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)^{-1} \mathrm{d}H_s \qquad (9)$$

where the time increment dt has been expressed using the change in the static head (or surface levels):

$$\mathrm{d}t = \frac{1}{Q}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)^{-1} \mathrm{d}H_s \qquad (10)$$

The reservoir free surface areas A_1 and A_2 can change with static head.

3. Calculus of Variations

In the optimal control of the parallel pumping process in Fig. 1, energy consumption in (8) is minimized and time in (9) is a constraint. Both are functionals of rotational speeds n_1 and n_2; that is, they depend on the rotational speed at every time instance of the process. Such an optimization problem involving functionals can be treated using the Calculus of variations, whose following basic theory can be found, for example, in [13] and is applied later on.

Consider minimizing the functional F under the equality constraint G:

$$F(y) = \int_a^b f(y(x))\mathrm{d}x \qquad (11)$$

$$G(y) = \int_a^b g(y(x))\mathrm{d}x - G_0 = 0 \qquad (12)$$

where f and g are known functions and y is the unknown function. The (Gateaux) variation of $F(y)$, which is equivalent to a conventional derivative in the analysis of functions, is

$$\delta F(y, \Delta y) = \lim_{\varepsilon \to 0} \frac{F(y+\varepsilon\Delta y) - F(y)}{\varepsilon} \qquad (13)$$

where Δy is an arbitrary function in the same interval as y. The minimum of $F(y)$ occurs when the variation is $\delta F(y;\Delta y)=0$ for every Δy. By applying this condition, we get the necessary condition for optimality:

$$\frac{\mathrm{d}f}{\mathrm{d}y} + C\frac{\mathrm{d}g}{\mathrm{d}y} = 0 \qquad (14)$$

It remains to find the function y and constant C such that (14) and (12) hold.

If functionals F and G (and functions f and g) depend on two unknown functions, y_1 and y_2, the above result of the constrained case becomes

$$\frac{\mathrm{d}f}{\mathrm{d}y_1} + C\frac{\mathrm{d}g}{\mathrm{d}y_1} = 0 \qquad (15a)$$

$$\frac{\mathrm{d}f}{\mathrm{d}y_2} + C\frac{\mathrm{d}g}{\mathrm{d}y_2} = 0 \qquad (15b)$$

Functions y_1 and y_2 are optimal if in addition to (15) the constraint $G(y_1,y_2)=0$ is satisfied.

4. New Control Procedure for Parallel Pumping

In continuous pumping processes. where the objective is to maintain a nearly constant head with a changing flow rate or a constant flow rate with a changing head, pumping time is not an important issue. On the other hand, in batch transfer systems, where a given amount of liquid is pumped from one tank to another on a regular basis, process time matters. In a representative pumping process in Fig. 1, the purpose is to fill the tank so that static head increases from $H_{s,min}$ to $H_{s,max}$. Pumping should be performed with minimum energy, while the process time is fixed at T_0. The rotational speeds of both pumps must be controlled optimally throughout the process. This optimization problem can be stated as

$$\begin{array}{ll} \text{find} & n_1(H_s), n_2(H_s), \quad H_{s,\min} \le H_s \le H_{s,\min} \\ \text{to minimize} & E(n_1, n_1) \\ \text{subject to} & T(n_1, n_1) - T_0 = 0 \end{array} \qquad (16)$$

The optimal control law for pump rotational speeds is obtained from (15), where f and g are replaced by the integrands of (8) and (9), and where y_1 and y_2 correspond to rotational speeds n_1 and n_2. These equations are equivalent to minimizing $(P+C)/Q$ throughout the process with a fixed value of C:

$$\begin{array}{ll} \text{find} & n_1, n_2 \\ \text{to minimize} & \dfrac{P+C}{Q} \\ \text{such that} & n_{\min} \le n_1 \le n_{\max},\ n_{\min} \le n_2 \le n_{\max} \end{array} \qquad (17)$$

where P and Q are the total power and flow rate of the pump pair. The minimization can be performed using, for example, the Sequential Quadratic Programming (SQP) method. The Euler-Lagrange multiplier C in (17) represents the sensitivity of energy consumption to process time. It is unknown together with pump rotational speeds. In practice, the optimal operation must be calculated several times to find the correct value of C and thus the correct process time. The numerical solution procedure is further explained below.

Several iterations are required to apply the optimal control law (17). In the innermost iteration loop, we solve the operation point from (1)-(2) and (5)-(6) when the pumps' rotational speeds are given. In the middle loop C is given, and optimal rotational speeds n_1 and n_2 are solved from (17) for sufficiently many static heads between $H_{s,min}$ and $H_{s,max}$. After this, total energy consumption and process time are obtained by numerical integration of (8) and (9). Finally in the outermost loop, we search for the value of C that gives T-T_0=0, that is, the correct process time. Next, we show a simple example of two parallel pumps controlled according to this algorithm.

5. Example

5.1. Initial values

In the example process in Fig. 3, two unequally sized parallel pumps, pump 1 and pump 2, pump 100 m^3 of water from a large reservoir (A_1=∞) into a tank with a cross sectional area of A_2=20 m^2. The static head at the start of pumping is $H_{s,min}$=2 m, and the pumping ends at $H_{s,max}$=7 m. The friction constant in (5) is K=2000 s^2/m^5, and the fixed target pumping time is T_0=1120 s.

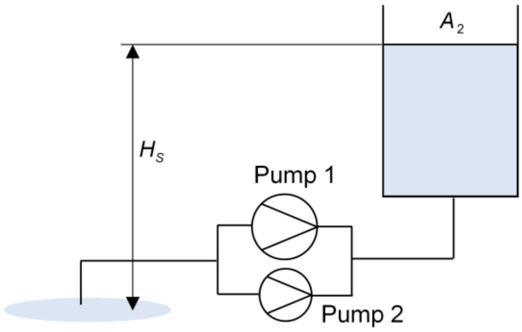

Fig. 3. Example of a parallel pumping system.

In this example, the reference rotational speeds are those corresponding to the head curves in Fig. 2: n_{1r}=1500 rpm and n_{2r}=1500 rpm. The reference head curves are quadratic functions:

$$H_{r1} = a_0 + a_1 Q_r + a_2 Q_r^2 \qquad (18)$$

$$H_{r2} = a_3 + a_4 Q_r + a_5 Q_r^2 \qquad (19)$$

where a_0=36 m, a_1=0, a_2=-6000 s^2/m^5, a_3=28 m, a_4=0, and a_5=-2200 s^2/m^5. By combining (1)-(2) and (18)-(19), the pump heads at rotational speeds n_1 and n_2 and flow rates Q_1 and Q_2 become

$$H_1 = a_0 \left(\frac{n_1}{n_{1r}} \right)^2 + a_1 Q_1 \left(\frac{n_1}{n_{1r}} \right) + a_2 Q_1^2 \qquad (20)$$

$$H_2 = a_3 \left(\frac{n_2}{n_{2r}} \right)^2 + a_4 Q_2 \left(\frac{n_2}{n_{2r}} \right) + a_5 Q_2^2 \qquad (21)$$

With the help of (20) and (21), the operating point, that is, the total flow rate Q and head H, can now be solved by iteration from (5)-(6). The pump efficiencies at reference rotational speed are also approximated with quadratic functions, which gives us:

$$\eta_1 = 1 - \left(1 - b_0 - b_1 Q_1 \left(\frac{n_{1r}}{n_1} \right) - b_2 Q_1^2 \left(\frac{n_{1r}}{n_1} \right)^2 \right) \left(\frac{n_{1r}}{n_1} \right)^k \qquad (22)$$

$$\eta_2 = 1 - \left(1 - b_3 - b_4 Q_2 \left(\frac{n_{2r}}{n_2} \right) - b_5 Q_2^2 \left(\frac{n_{2r}}{n_2} \right)^2 \right) \left(\frac{n_{2r}}{n_2} \right)^k \qquad (23)$$

where b_0=0.14, b_1=80, b_2=-350, b_3=0.15, b_4=35, and b_5=-500. The constant k in (22) and (23) is 0.15 for both pumps. The pump power required in (17) is calculated from (7) using the above head and efficiency curves. We note that at low rotational speeds, that is, n<0.7n_r, pump efficiency drops considerably, and (22)-(23) do not hold.

5.2. Results

As the surface levels change in the tanks, also optimal rotational speeds and flow rates change. The optimal path of the operating point is calculated in 20 different static heads spaced evenly between initial $H_{s,min}$=2 m and final $H_{s,max}$=7 m values using the procedure developed in Section 4. Process energy consumption E=63.9 MJ and time were obtained by numerically integrating (8) and (9) over the found operating points. The value of C=41310 W corresponds to the desired process time of T=1120 s.

The individual flow rates of pumps 1 and 2 during the optimal process are shown in Fig. 4. The system flow rate, which is the sum of the flow rates of the parallel pumps, can be seen to connect the initial and final system curves. The flow rate of pump 1 decreases and that of pump 2 increases slightly during the process; as a result, the total flow rate remains almost constant. In fact, keeping the flow rates of both pumps constant during the process does not significantly increase the energy consumption, as observed also in case of a single pump [8]. However, this control is not possible if either pump reaches its maximum allowed rotational speed. It is also possible that one of the pumps must be shut down because of too high a static head or started only after the other pump has been running for a while. We did not take these conditions into account.

Figure 5 shows the optimal rotational speeds as a function of static head. The rotational speed of pump 2 increases almost linearly from 1308 rpm to 1453 rpm while pump 1 operates at a higher speed. In this example, the time limit 1120 s is so strict that pump 1 reaches the maximum rotational speed limit of 1500 rpm when static head is 5.4 m. To minimize the process time, both pumps must run at the maximum allowed speed (1500 rpm) throughout the process. This process takes 1047 s time and requires 68.4 MJ of energy.

In the power curve method, the pump flow rate Q and head H are calculated from the affinity laws (1)-(3) using the values of power P and rotational speed n which are obtained from VSD. Flow or pressure measurements from the system are not necessary. For a single pump, minimum specific energy control can be implemented in a VSD by varying rotational speed in small steps dn to minimize P/Q [5]. With a time limit, the objective function is $(P+C)/Q$, where C is also unknown [8]. With two parallel pumps, there are three unknowns: n_1, n_2, and C, see (17). However, it may not be possible to optimize using a VSD system without measuring the system head. The power estimated by a VSD fluctuates, and the mean value can be in error by 2 %. Furthermore, the function $(P+C)/Q$ is very flat around its minimal points; in our example a 2 % change in $(P+C)/Q$ corresponds to more than a 100-rpm change in rotational speeds.

The flow rate of a pump can be obtained also from the pump head curve using a measured pressure difference over the pump. This is known as the head curve method. Even though the estimation accuracy can be improved by combining the information from head and power curve methods, it might not be enough to reliably minimize $(P+C)/Q$. We believe that the best way to implement the proposed control law is to calculate optimal rotational speeds as a function of static head, as presented in this paper, and program rotational speed directly into the VSD as a function of process time or measured static head. However, the system curve must be known to perform the calculations.

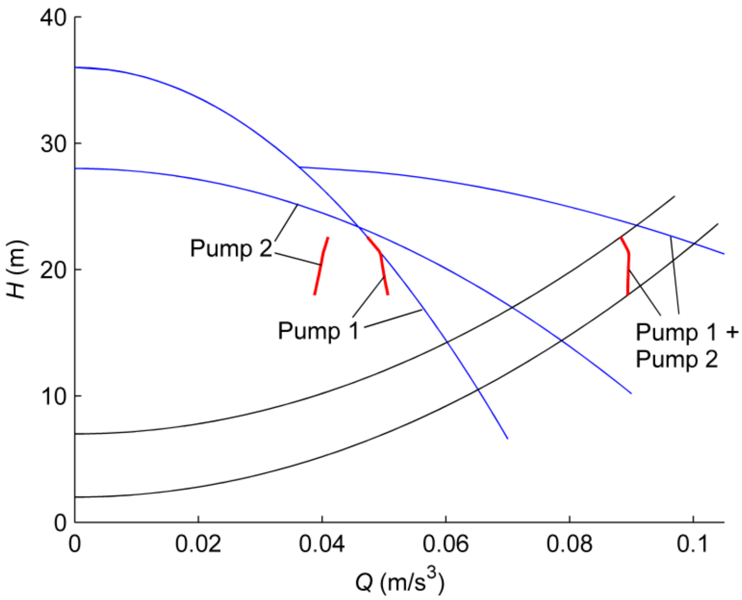

Fig. 4. Optimal flow rate and head of parallel pumps in an example reservoir filling process. The system curves correspond to the low ($H_{s,min}$) and high ($H_{s,max}$) surface levels.

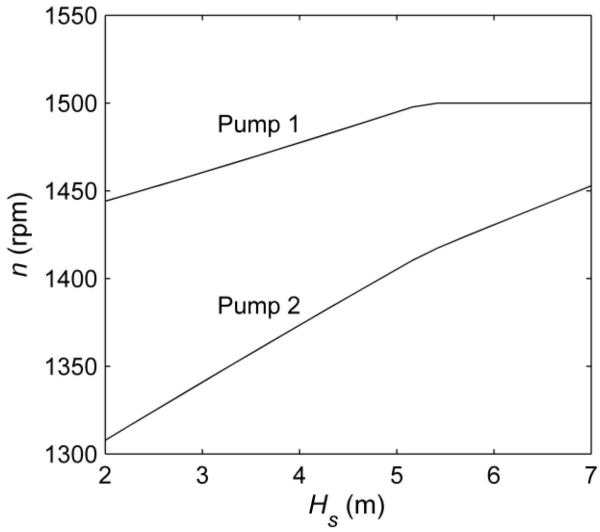

Fig. 5. Optimal rotational speeds of parallel pumps as a function of surface level (static head).

6. Conclusions

To address the problem of high energy consumption in pumping systems, we present an optimal control scheme for operating parallel pumps. This scheme is intended for batch-wise processes, for instance, for filling or emptying a tank in a waste water process, where process time is limited. We have presented an analytical control law (17) that gives optimal rotational speeds individually for both pumps throughout the process. The results calculated from the algorithm can be implemented in modern VSD systems that allow sensorless estimation of the pump's operational state.

A calculation example shows that the optimal flow rates of the pumps are almost constant during the pumping process. Because the objective function in the control law is very flat around its minimum, almost minimal energy consumption can be achieved by maintaining constant flow rates for both pumps throughout the process. However, this simple scheme is not possible if either pump must be shut down or run at the maximum allowed rotational speed to satisfy the process time limit.

Because of the flatness of the objective function, large changes in the pumps' rotational speeds lead to only small changes in energy consumption. This is not desirable in sensorless pump control, since the rotational speeds would have to be varied in large steps to noticeable change the specific energy. To guarantee smooth operation, we recommend that optimal rotational speeds be numerically calculated in advance and programmed in the VSD. Soft pump starts, smooth operation, and high efficiency all promote high pump reliability. High pump reliability can be as equally an important criterion as low energy consumption, because the costs of pump failure can well exceed those of pump operation.

The optimal control law for parallel pumps was derived based on a corresponding result for a single pump. This analysis might be further extended to two unequally sized pumps operating in *series*. Analysis of pumps in series combined with the present results could allow optimization of a more complex pumping system in terms of energy and time.

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Nomenclature

A_1, A_2 Free surface areas, m^2

VSD Variable Speed Drive

BEP Best Efficiency Point
C Euler-Lagrange multiplier, (17), W
E Energy consumption of pumping process, (8), J
g Gravitational acceleration, 9.81 m/s^2
H Total head, (1b) and (4), m
H_s Static head, m
$H_{s,min}$ Static head at the beginning of process, m
$H_{s,max}$ Static head at the end of process, m
k Constant in (4)
K Frictional constant in (5), s^2/m^5
n Rotational speed, rpm
P Pump power, (7), W
Q Flow rate, m^3/s
t Time, s
T Time of pumping process, (9), s
T_0 Target time for pumping process, s
V Volume of fluid pumped, m^3

Greek Letters
ρ Density of fluid, kg/m^3
η Pump efficiency

Subscripts
r Reference point

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