Optimization of ship speed profile along a route under variable weather conditions

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Abstract:

The problem of ship speed optimization under specified weather conditions, using dynamic optimization methods, is tackled in this article. Analytical models based on the literature are developed in order to simulate the vessel's resistance and propulsion. Then, four ship speed dynamic optimization problems with increasing complexity are described and stated mathematically. The objective is to find the optimal ship speed profile, along a specified route with known weather conditions, which minimizes fuel consumption. The problems are solved by dynamic optimization software. The optimization results indicate that significant reduction in fuel consumption can be achieved with respect to profiles specified empirically or arbitrarily, especially in the cases of variable travel duration. The work presented here is a first step towards the optimization of the energy system of a ship that will cover all loads (propulsion, electrical and thermal energy) under variable conditions.

Keywords:

Ship Speed Optimization, Dynamic Optimization, Weather Routing, Ship Energy System.

1. Introduction

Seaborne transportation is one of the main transportation modes and maybe the only cost effective option for transporting large volumes of cargo between continents. Approximately eight billion tons of goods are carried by sea each year. However in recent years, due to increased fuel prices, depressed market conditions and environmental issues regarding air emissions from ships, the need for optimizing ship operation, especially in the domain of fuel consumption, has become a necessity rather than simply a modern trend. For this purpose, methods for more and more accurate estimation of ship resistance and propulsion power are developed, and a holistic approach is followed for the design and operation of a ship.

One particular aspect of operation optimization is the determination of the optimal route that a ship should take in order to go from port A to port B, taking into consideration the weather encountered along alternative routes. The objective of this optimization can be the minimization of fuel consumption under constraints related to the time of arrival, the safety of the ship, etc. The related field of research is known with the name *Weather Routing* [1-9].

Even with a predetermined route, there is still room for optimization: the optimal speed profile is requested, i.e. the speed as a function of space and time that satisfies an objective, e.g. the speed profile that minimizes the fuel consumption. The fact that the weather changes with space and time makes the optimization problem inherently dynamic and its solution is far from trivial. A contribution to this field is attempted with the present work.

2. Description of the Ship Speed Optimization Problems Under Study

The problem of ship speed optimization addressed in this work, can be stated with the following question:

"*What is the optimal speed profile a certain ship must follow along a specified route with variable weather conditions, in order to minimize fuel consumption while satisfying certain safety and regulatory constraints?*"

This is an inherently dynamic optimization problem, because of at least two reasons:

- The weather profile that the ship encounters during her voyage is, generally, time and space dependent. Thus, time dependency unavoidably appears in all variables of the problem such as speed, ship resistance and required propulsive engine power.
- Of course, the time dependency does not necessarily characterize an optimization problem as dynamic. There is also the issue of interdependency between time intervals. Even if the time horizon of the trip is divided into distinct periods (time intervals) of steady state operation, the fact that the route is specified introduces such interdependency. Thus, the optimal speed in each and every interval depends on the weather conditions of both the current interval and of every other interval.

In this work by the term "weather conditions" (referred to as weather profile or weather state also), only the wind and sea waves are taken into account, while other weather characteristics such as temperature, pressure, cloud, rain or fog are not considered. So, the term weather profile implies the wind speed and direction as well as the wave-height and direction at any point in time and space along the route of the ship.

This study treats the weather deterministically under the assumption that it can be predicted with sufficient accuracy at every point in time and space. In a further development of the work, stochastic models could be used, in order to model and describe weather conditions more realistically. It is true that, no matter how accurately the models utilized can calculate ship resistance or no matter how efficiently the dynamic optimization method solves the problem, the final optimal solution is as accurate as the weather prediction.

Starting from the statement of the ship speed optimization problem written at the beginning of this section, four distinct dynamic optimization example problems are formulated, described and mathematically stated in the rest of this section. The key idea is to present ship speed optimization problems of increasing complexity by varying the assumptions regarding the final time (fixed or variable) and the weather state (time-dependent or space and time-dependent). The properties that differentiate the four problems from each other are presented in Table 1.

Table 1. Main characteristics of the four problems.

Problem	Weather as a function of	Duration of trip
1	time	fixed
2	time	control variable
3	space and time	fixed
4	space and time	control variable

2.1. Common characteristics of the four problems

In all four problems the distance to be traveled by the ship is predetermined and the objective of optimization is the minimization of fuel consumption. The ship speed is a control variable in all problems (alternatively, the propulsion engine brake power could have also been selected as a control variable instead of speed, since they are related by a one to one relation). In problems 2 and 4, the duration of the trip is also a control variable.

While traveling, the ship encounters several resistances such as calm water resistance, added wind resistance and added wave resistance, as well as secondary resistances such as appendage resistance, bulbous bow resistance, etc. Calm water resistance is a function of ship speed and ship characteristics (geometry, hull characteristics). Added wind resistance is a function of ship characteristics, ship speed and wind speed that, in turn, is determined by the weather forecast. Added wave resistance is also dependent on the weather forecast, since it is a function of ship speed, ship characteristics and wave height that is, in turn, determined by wind speed; thus, it is also a function of the weather conditions. Models for the calm water resistance as well as the added wind and wave resistances have been developed using information from the literature [10-30]. The total resistance is calculated with the function

$$R_T = R_T(\mathbf{p}, V, \mathbf{WS}) \quad (1)$$

where

V speed of the ship

$\mathbf{p}$ vector that denotes the time independent characteristics of the ship and the hull

$\mathbf{WS}$ weather state.

The weather state is defined by the equation

$$\mathbf{WS} = (U_{wind}, \psi_{wind}, H_s, \theta_{waves}) \quad (2)$$

where

U_{wind} wind speed

ψ_{wind} wind direction

H_s significant wave height defined as the mean wave height (trough to crest) of the highest third of the waves

θ_{waves} wave direction.

Thus, in order to fully describe the weather state and use it as input to the problems, four parameters must be known: wind speed, wind direction, significant wave height and wave direction. However, the significant wave height can indeed be deduced, by interpolation, from the wind speed using the Beaufort scale data. Furthermore, for simplicity here, the wind and wave are always assumed to be in the heading direction of the ship, i.e.

$$\begin{aligned} \psi_{wind}(t) &= 0.0 \\ \theta_{waves}(t) &= 0.0 \end{aligned} \quad (3)$$

Thus, the weather state for the whole trip is adequately described by providing the wind speed profile, U_{wind}, as a function of time or space and time.

Furthermore, suitable mathematical models are used for the required shaft power as well as the coupling of resistance and propulsion based on information from the literature [10,11,18,28].

The corresponding effective power (or towing power), necessary to tow the ship through the water, at speed V in absence of propulsive power, is given by the equation

$$\dot{W}_e = R_T \cdot V \quad (4)$$

The engine brake power is given by the equation

$$\dot{W}_b = \frac{\dot{W}_e}{\eta_{tot}} \quad (5)$$

where η_{tot} is the total propulsive efficiency, which is calculated by means of an analytic model not presented here due to space limitations.

2.2. Description and mathematical statement of example problem 1

The dynamic optimization problem can be mathematically stated as a minimization problem using a Differential − Algebraic Equations (DAE) formulation. In this simple case, one control variable is used, the speed of the ship, V, while the time of arrival (final time), t_f, is known. Since the goal is the minimization of the fuel consumption, m_f, the objective is stated as

$$\min_{V} m_f = \int_0^{t_f} b_f \cdot \dot{W}_b \cdot dt \qquad (6)$$

where b_f is the specific fuel oil consumption (SFOC) of the engine which, for a specific engine, is a function of the brake power or equivalently of the engine load factor f_L

$$b_f = b_f(\dot{W}_b) \text{ or } b_f = b_f(f_L) \qquad (7)$$

$$f_L = \frac{\dot{W}_b}{\dot{W}_{b_n}} \qquad (8)$$

where W_{b_n} is the maximum continuous rating of the engine.

The wind speed is a function of time along the route:

$$U_{wind} = U_{wind}(t) \qquad (9)$$

where t is the elapsed time. Of course, the distance travelled, d, the elapsed time and the ship speed are inter-connected by the equation

$$d = \int_0^{t} V \cdot dt \qquad (10)$$

In order to conclude the mathematical statement of the problem, the necessary boundaries on the variables as well as the initial and final points, if they exist, of the differential variables are included.

For example, limits are imposed on the speed of the vessel

$$V_{\min} \leq V \leq V_{\max} \qquad (11)$$

and on the load factor of the engine

$$f_{L_{\min}} \leq f_L \leq f_{L_{\max}} \qquad (12)$$

There may be need of additional inequality constraints, but they are not written here, for brevity. For the two differential variables, distance travelled and fuel consumption, the initial points are known

$$\begin{aligned} d(0) &= 0 \\ m_f(0) &= 0 \end{aligned} \qquad (13)$$

while the final point is known for the distance travelled only:

$$d(t_f) = d_{final} \qquad (14)$$

2.3. Description and mathematical statement of example problem 2

The objective function in this case is a function of two control variables:

$$\min_{t_f, V} m_f = \int_0^{t_f} b_f \cdot \dot{W}_b \cdot dt \qquad (15)$$

The resistance − propulsion interconnection model is again described by (1)−(5) and (7)−(14) with the addition of higher and lower bounds on the duration of the trip:

$$t_{f_{\min}} \leq t_f \leq t_{f_{\max}} \qquad (16)$$

2.4. Description and mathematical statement of example problem 3

In Example Problem 3, a further complication is introduced: the duration of travel is fixed, but the weather state is considered as a function of both space and time. Thus, the problem is stated mathematically by (1)–(14), with (9) being replaced by the equation

$$U_{wind} = U_{wind}(t, d) \quad (17)$$

2.5. Description and mathematical statement of example problem 4

Example problem 4 is the most complex of the four optimization problems. The problem is stated by (15) as the objective function, (1)–(5) and (7)−(14) for the resistance−propulsion interconnection model, while (9) is replaced by (17).

3. Simulation and Optimization Software

The same ship will be used in all four problems with the following basic dimensions:

$$L = 124 \text{ m} \qquad B = 17 \text{ m} \qquad T = 7 \text{ m} \qquad \nabla = 1200 \text{ m}^3$$

Considering the ship propulsion, a suitable marine Diesel engine is selected with a target design speed of 20 knots. A quick simulation based on the model of calm water resistance is initially performed, which results in a required Maximum Continuous Rating (MCR) of 8692 kW. Using the engine selection software from a manufacturer's website, a two stroke heavy fuel Diesel engine is selected with MCR of 8692 kW at 127 RPM.

Furthermore, the specific fuel oil consumption is given in Fig. 1 as a function of the engine load factor, f_L. For the computer calculations, an analytic function is needed, which is obtained by interpolation based on the data of Fig. 1.

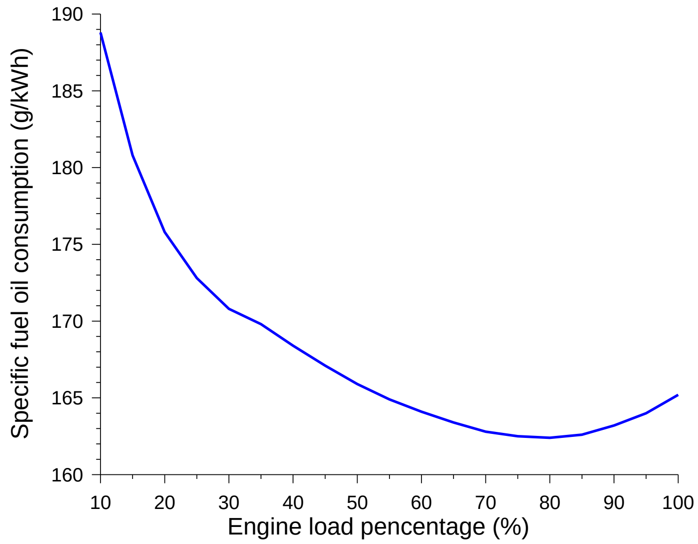

Figure 1. Specific fuel oil consumption as a function of the engine load.

Complex dynamic models are used in order to calculate ship resistance and propulsion power taking into consideration the effects of weather conditions. The related simulation and solution of the dynamic optimization problems is performed by means of the gPROMS software [31].

Considering the solution approach, two dynamic optimization methods to solve complex non-linear dynamic problems are available in the software: the single-shooting and the multiple-shooting method. The single-shooting method is suitable for problems with many state variables, few control variables and few control intervals; it is the solution method applied in this study. The technique uses Control Vector Parameterization (CVP), where time-varying controls are defined as simple functions of time over a number of control intervals. In this study, the discretization of the controls is selected to be piecewise constant in each interval. Further details about the single shooting method can be found in the literature [32,33].

4. Solution of the Optimization Problems

4.1.Numerics of optimization

The numerical values of the parameters appearing in each of the four optimization problems are summarized in Table 2.

Table 2. Values of parameters for the four example problems

Parameter	Problem 1	Problem 2	Problem 3	Problem 4
Ship speed upper limit	20 kn	20 kn	20 kn	20 kn
Ship speed lower limit	0 kn	0 kn	0 kn	0 kn
Distance to be travelled	400 km	400 km	400 km	400 km
Trip duration	15 h	control variable	15 h	control variable
Trip duration lower limit	--	15 h	--	15 h
Trip duration upper limit	--	25 h	--	25 h
Number of time intervals	15	25	15	25
Length of time intervals	1 h	control variable	1 h	control variable
Number of space intervals	--	--	8	8
Length of space intervals	--	--	50 km	50 km

4.2. Solution of example problem 1

In example problem 1, it is considered that the weather state is a function of time only, while it is uniform over space. A weather profile (Fig. 2(a)) that represents a "storm" at approximately the middle of the trip duration is considered. The optimal ship speed profile is shown in Fig. 2(b).

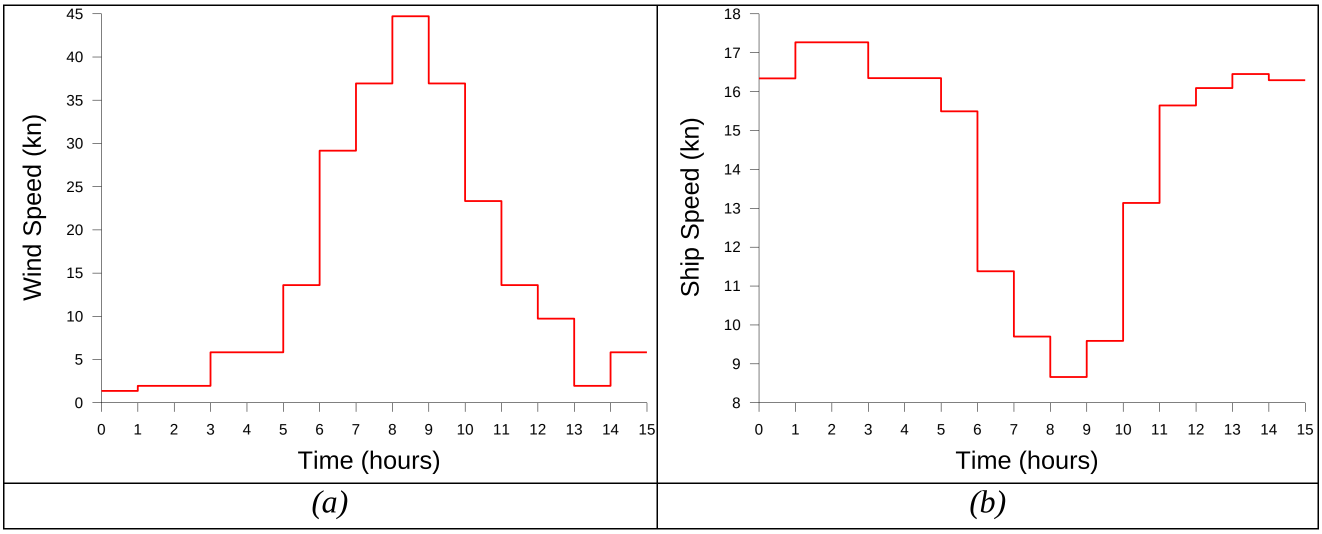

Figure 2. (a) Wind speed and (b) optimal ship speed versus time for problem 1.

The optimal fuel oil consumption is found equal to 10.289 tonnes. The optimization was concluded in 271 seconds, performing 10 major NLP iterations at an Intel® Core™2 Quad Processor Q9650 cpu at 3GHz with 8Gb of RAM.

Since a rigorous proof that this is indeed the optimal profile is not possible, three additional possible speed profiles are selected arbitrarily, Fig. 3, and the results are compared with those of the optimal speed profile.

Case A corresponds to the optimal speed profile. Case B considers a steady speed of 14.4 kn. Cases C and D are constructed based on the idea of slow steaming. Thus, the ship is set to travel through the "bad weather", which, based on Fig. 3, lasts in the time interval of 6 to 11 hours, with a low speed. In case C, the ship travels through the bad weather with a speed of 5.83 kn, while in case D that speed is set at 11.66 kn. For the rest of the trip in both cases the speed is set as near as possible to the nominal speed of 20 kns, adapted accordingly so as to ensure that the ship will cover the 400 km distance over exactly 15 hours.

The corresponding values of fuel consumption are the following:

$m^*_{f,A} = 10.289$ ton $\quad m_{f,B} = 11.212$ ton $\quad m_{f,C} = 11.455$ ton $\quad m_{f,D} = 10.483$ ton

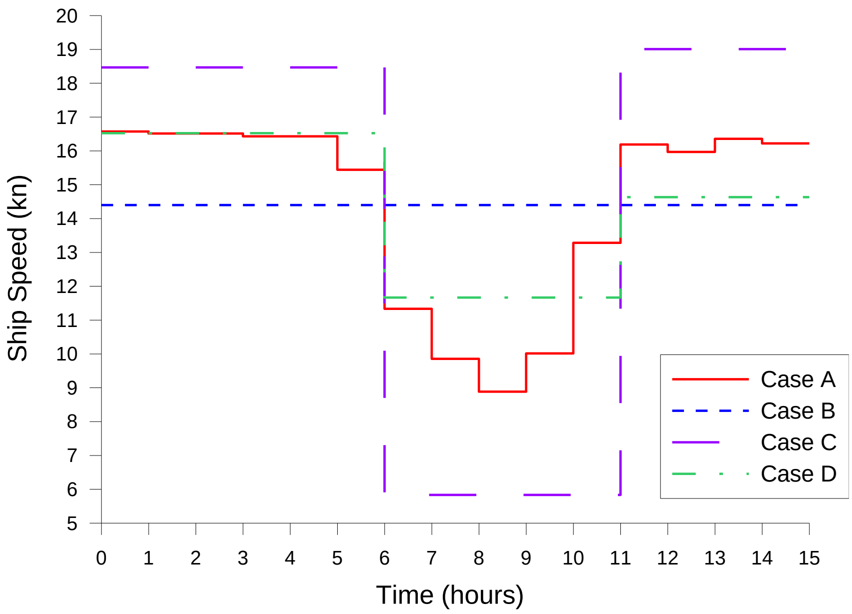

Figure 3. Alternative speed profiles for problem 1.

It is noted that the speed profile resulted from optimization is indeed better compared to the three alternative profiles. Specifically, when compared with profile B, which corresponds to performing the trip with an average steady speed, the optimal speed profile yields an improvement of approximately 9% in terms of fuel required. Furthermore, when compared with profile C, which suggests travelling through the bad weather with a very low speed (5.83 kns), the improvement is even better, approximately 11%. This is expected since, with a fixed trip duration, the ship must increase its average speed in the time periods outside the bad weather, in order to reach the destination on time. A scenario close to optimal is represented by profile D, where the ship travels through the bad weather with a speed of approximately 12 kn, which is on average a little more than what the optimal solution suggests.

4.3. Solution of example problem 2

In problem 2, the forecast of the weather state profile has to be provided up to the upper bound of the trip duration, in contrast to exactly 15 hours specified in problem 1. Figure 4(a) provides such a weather profile as input, which in essence is an extension of the profile selected in Fig. 2(a) for problem 1. The exact values of wind speed are selected arbitrarily, but the key idea behind their selection was to create another time period of very bad weather after the 15th hour. Thus, the question arises as to whether the ship should follow the previous optimal solution and arrive at her destination in 15 hours or it is better to travel also through the second storm, in order to gain extra travelling time and decrease the average speed and thus fuel consumption. The weather state profile is considered again uniform over space.

The resulting optimal ship speed profile is given in Fig. 4(b). The optimization was concluded in 364 seconds, performing 7 major NLP iterations at a an Intel® Core™2 Quad Processor Q9650 cpu at 3GHz with 8Gb of RAM.

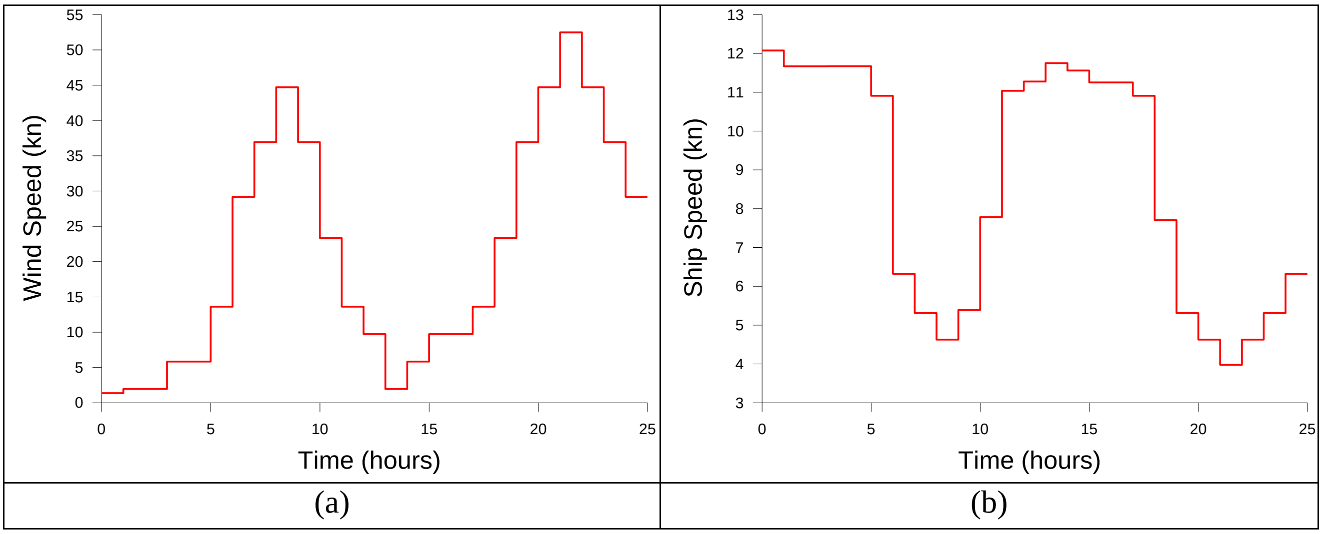

Figure 4. (a) Wind speed and (b) optimal ship speed versus time for problem 2.

The optimal fuel oil consumption is found equal to 6.115 ton. The optimizer selects the upper bound (25 hours) of the time horizon as the optimal trip duration.

As an indication that the time horizon and the speed profile determined are optimal, four more dynamic optimizations are performed with − this time− fixed trip durations of 16, 18, 20 and 23 hours. The results shown in Table 3 verify that the optimal choice for the trip duration is indeed the upper bound.

Table 3. Results for various trip durations in Problem 2

Trip Duration (hours)	Fuel Consumption (ton)
25	6.115
23	6.457
20	6.856
18	7.508
16	9.175
15 (problem 1)	10.289

The results suggest that there is no need for the ship to avoid travelling into the second bad weather region; instead, she should take advantage of the extra 10 hours of travelling time, in order to decrease significantly the average required speed. More specifically, a remarkable decrease of 40.5% in fuel consumption is observed, in comparison with the case of 15 hour duration of the previous example.

4.4. Solution of example problem 3

In problem 3, the complexity of the dynamic optimization problem is increased, because the weather state profile is a function of both space (distance travelled) and time (trip duration). Various possible weather profiles, which produce interesting speed optimization scenarios, can be selected as an input to the problem. The weather profile selected in this study is given with the help of 3-D plots (surfaces) of wind speed versus time and space (Fig. 5). The key idea behind its selection is to have a storm that initiates (t = 0) near Port B and is heading to Port A as time passes, while the ship travels on the opposite direction from Port A to Port B. Interesting questions may arise considering the speed strategy the ship will eventually follow: e.g. should the ship speed up at the beginning of the journey, thus increasing the average speed, so as to take advantage of the good weather regions at start and meet the storm as near port B as possible or it is better to keep a small average speed throughout the trip and let the storm come towards her along the way?

It is noted that in this case along with time, space is discretized also. Thus, both space and time intervals are defined. The optimal speed profile is given in Fig. 6(a).

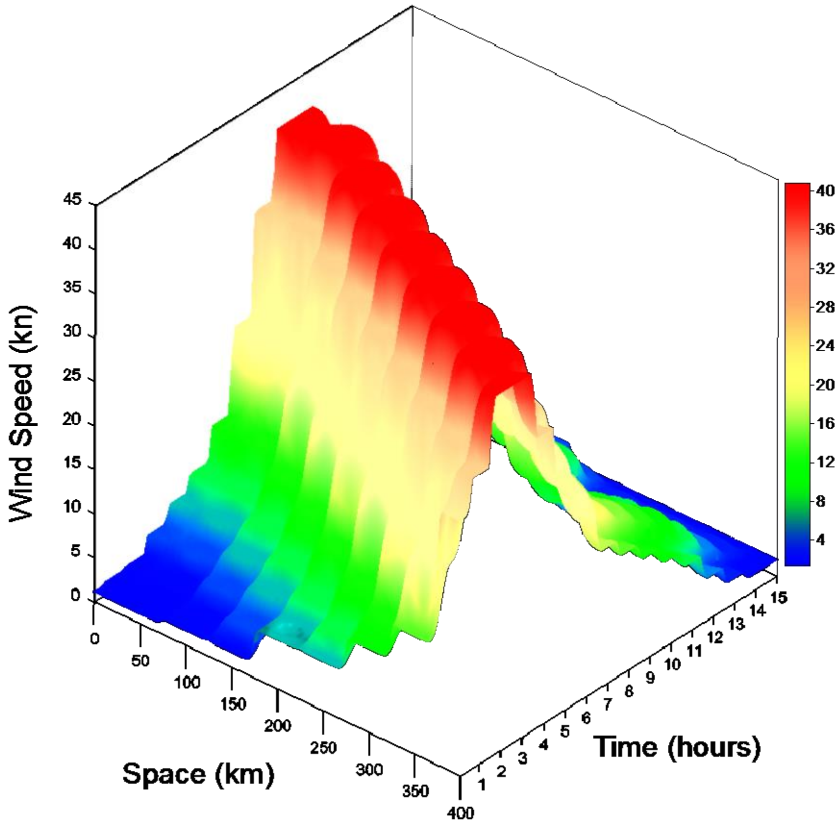

Figure 5. Wind speed versus time and space for problem 3.

The optimal fuel oil consumption is found equal to 9.787 ton. The optimization was concluded in 512 seconds, performing 15 major NLP iterations at an Intel® Core™2 Quad Processor Q9650 cpu at 3GHz with 8Gb of RAM.

Since, again, a rigorous proof that this is indeed the optimal profile is not possible, three additional possible speed profiles are selected arbitrarily, Fig. 6(b), and the results are compared with those of the optimal speed profile.

Case A corresponds to the optimal speed profile. Case B considers a steady speed of 14.4 kn along the route. Case C is constructed upon the idea that the ship travels at top speed for the first 5 hours, then based on the wind speed profile, Fig. 6(b), she reaches the storm, travels through it with low speed for 4 hours, and then concludes the journey with an appropriate speed to reach port B within the 15 hour horizon. Finally for the last scenario, Case D, we assume that the ship travels with a constant speed of 12 kn (little more than half its top speed) for the first 9 hours, until she reaches and passes through the storm, and then speeds up accordingly to reach her destination. The corresponding values of fuel consumption are the following:

$$m^*_{f,A} = 9.787 \text{ ton} \qquad m_{f,B} = 10.536 \text{ ton} \qquad m_{f,C} = 12.022 \text{ ton} \qquad m_{f,D} = 10.551 \text{ ton}$$

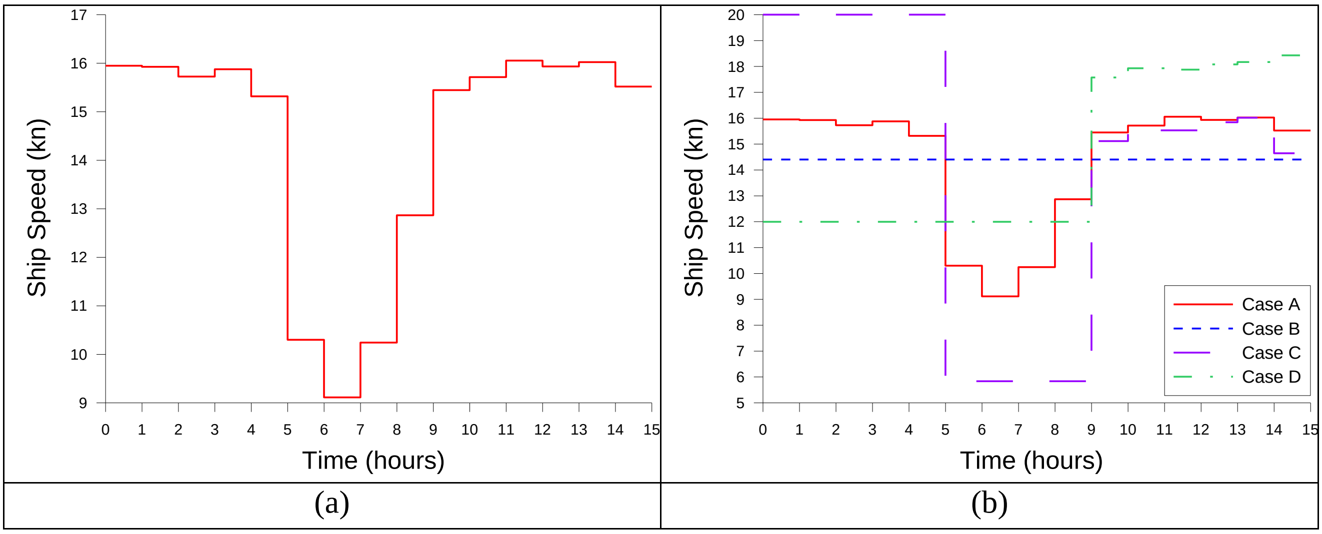

Figure 6. (a) Optimal ship speed and (b) possible speed profiles for problem 3.

It is noted that the speed profile resulted from optimization is indeed better compared to the three alternative profiles. When compared with profile B, an improvement of 7% in terms of fuel required is observed. Also, the plan of travelling towards the storm with top speed and then passing through the storm with a low speed (profile C) is proven to be a bad idea, since it costs approximately 28% more than the optimal solution. Finally, profile C bears little difference to profile B, when compared to the optimal solution.

4.5. Solution of example problem 4

Problem 4 combines the characteristics of all previous problems. As in problem 2, the forecast of the weather state profile is provided up to the upper bound of the time horizon in contrast to exactly 15 hours as previously. Such a profile is given again with the help of 3-D plots (surfaces) of wind speed versus time and space, Fig. 7, and is in essence an extension of the profile selected in Fig. 5 for problem 3. The key idea behind the creation of the specific profile was this time to have not one, but two sequential storms heading from port B to port A, while the ship travels from port A to port B. As in problem 2, the interesting question arises, whether it is better for the ship to reach port B within the 15 hour margin (as in problem 3) and encounter only the beginning of the second storm or to travel completely through the second storm as well in order to gain extra travelling time and decrease the average speed and thus the fuel consumption. The optimal ship speed profile is presented in Fig. 8.

The optimal fuel oil consumption is found equal to 5.893 ton. The optimizer selects the upper bound (25 hours) of the time horizon as the optimal duration of travel. The optimization was concluded in 689 seconds, performing 13 major NLP iterations at a an Intel® Core™2 Quad Processor Q9650 cpu at 3GHz with 8Gb of RAM.

Similarly to problem 2, as an indication that the time horizon and the speed profile determined are optimal, five more dynamic optimizations are performed with − this time− fixed trip durations of 15, 16, 18, 20 and 23 hours (Table 4).

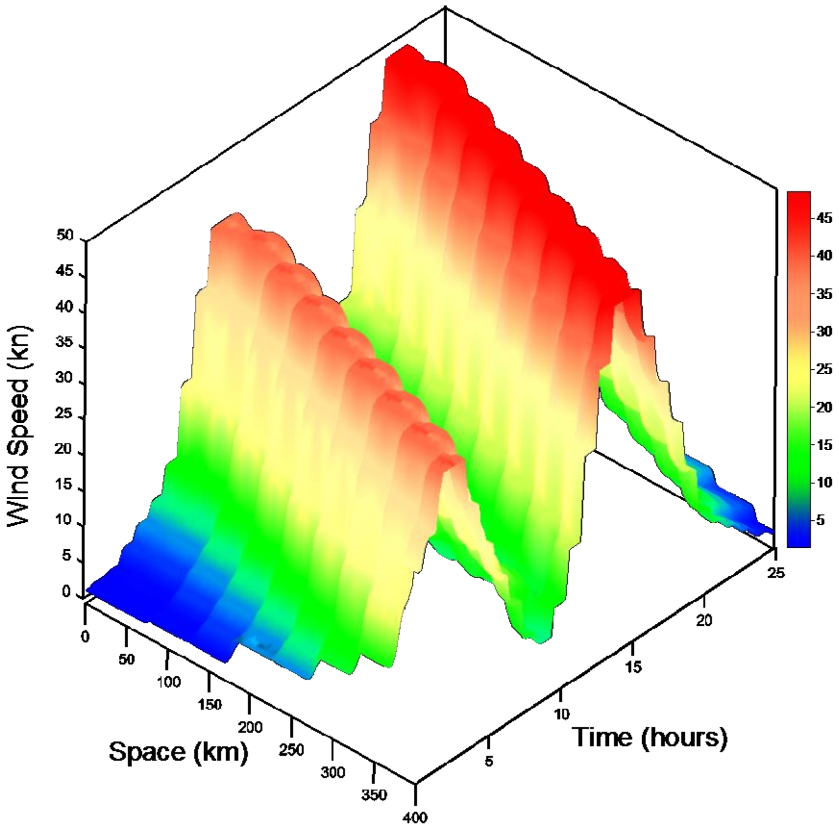

Figure 7. Wind speed versus time and space for problem 4.

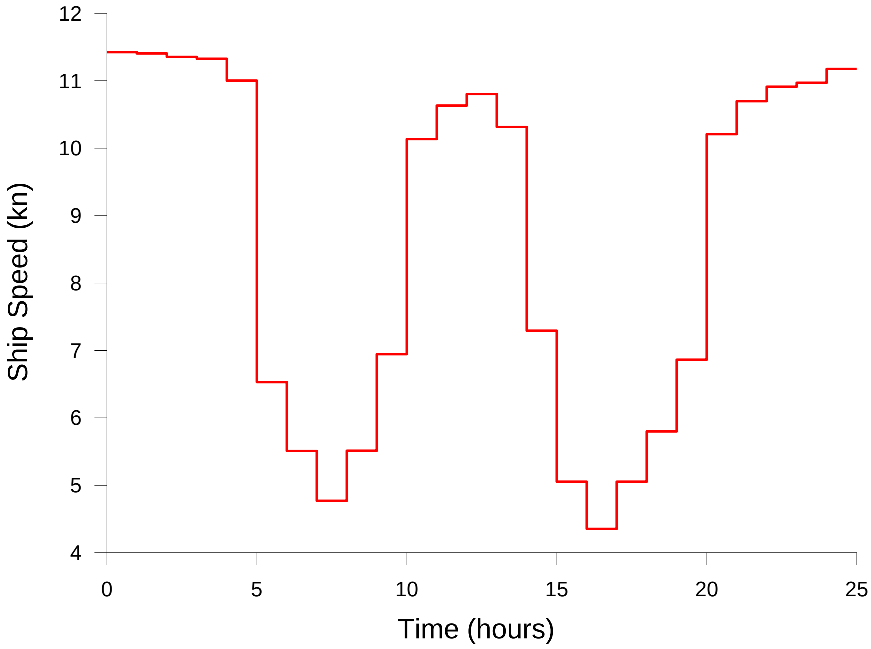

Figure 8. Optimal ship speed versus time for problem 4.

It is noted that the 15 h optimization depicted in Table 4 is not the same as the previous optimization solved in problem 3. In problem 3 after the ship encounters and passes through the storm, it experiences ideal weather until it reaches port B. However, this is not the case in the current problem. Even if the time horizon is kept in the lower limit of 15 hours, the ship cannot completely avoid the second storm and while reaching port B she will experience at least the beginning of the second storm. This alters dramatically the optimal result, since a 23.6% increase in fuel consumption is observed. The results prove that the true optimal choice for the trip duration is indeed the upper bound.

Table 4. Results for different time horizon values in problem 4

Trip Duration (hours)	Fuel Consumption (ton)
25	5.893
23	6.816
20	8.417
18	9.895
16	11.300
15	12.095

Once again, the ship should take advantage of the extra 10 hours of travelling time, in order to drastically decrease the average required speed and the fuel consumption. Furthermore, the idea of passing through the first storm quickly and encountering only a small portion of the second storm (e.g. setting the time horizon to its lower limit) fails completely, since it nearly doubles the fuel consumption, when compared to the optimal 25 hour trip duration.

5. Comments and Conclusions

In addition to remarks written in Section 4 after the solution of each problem, a few general comments are written here.

An observation derived from the solutions of all four problems is that the value for the optimal engine load factor remains low (below 70%). Especially in problems 2 and 4, where the ship travels for 25 hours with low speed, the Diesel engine operates with a load factor in the region of 10% to 20% in certain intervals. This is possible only because a state of the art Diesel engine is considered here, which can operate in such low load factors. Furthermore, the SFOC curve (Fig. 1) is rather smooth with 162.4 g/kWh at 80% load factor and only increasing to 175.8 g/kWh in the case of 20% load factor. However, when dealing with older or degraded engines, the SFOC may increase significantly at low load factors, which may alter significantly the optimal solution.

With the current work, it has been demonstrated that the optimal speed profile of a ship in a variety of conditions can be determined effectively by means of dynamic optimization methods and related software. This is a first step towards the optimization of the energy system of a ship that will cover all loads (propulsion, electrical and thermal energy) under variable conditions.

Nomenclature

B	breadth of the ship, m
b_f	specific fuel oil consumption (SFOC) of the engine, g/kWh
d	distance travelled, km
d_{final}	total distance travelled, km
f_L	engine load factor
$f_{L_{\min}}$	lower bound for the engine load factor
$f_{L_{\max}}$	upper bound for the engine load factor
L	overall ship length, m
m_f	fuel consumption, kg

p	vector of time independent characteristics of the ship
R_T	total resistance, kN
T	draught of the ship, m
$t_{f_{\min}}$	lower bound for the final time
$t_{f_{\max}}$	upper bound for the final time
t_f	final time, h
t	elapsed time, h
U_{wind}	wind speed
V	ship speed, kn
$V_{\min}$	lower bound for the ship speed, kn
$V_{\max}$	upper bound for the ship speed, kn
$\dot{W}_e$	effective power, kW
W_{b_n}	maximum continuous rating of the engine, kW
$\dot{W}_b$	brake Power of the engine, kW
WS	weather state
∇	displacement of the ship, m^3

Greek symbols

ψ_{wind}	wind direction
θ_{waves}	wave direction

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